



Spin-splitting in Rashba-Active Weak Links

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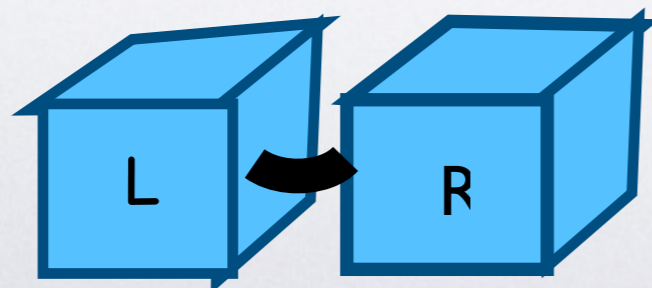


**Ministry of Science
and Technology**



PAZY
EXCELLING IN SCIENCE

הקרן
הלאומית
למדע



spin-orbit interaction
confined to the weak link



Use of electrical currents (or fields) to generate spin current and polarization **without** magnetic fields or ferromagnets



* spins of mobile electrons can be manipulated by spin-orbit interactions –the spin of an electron moving through a spin-orbit active material (e.g., semiconductor heterostructures) rotates around an effective magnetic field generated by the spin-orbit interaction.

רבי שלמה בן אברהם

E Rashba



* Rashba spin-orbit interaction is significant at surfaces and interfaces due to strong internal uncompensated atomic electric fields (normal to surface) that appear since the surface potential breaks the symmetry of the atomic orbitals there (strong atomic fields no longer cancel as they do in the bulk). Electric fields generated by gates can then modulate the strength of the Rashba interaction.



*tunneling amplitude with spin-orbit interaction

and spin splitting by Rashba interaction

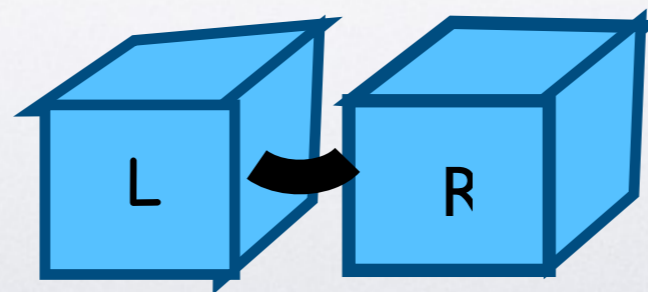
*Rashba splitting of Cooper pairs

*breaking time-reversal symmetry

—by a Zeeman field

—by an AC Rashba interaction created by a slowly-rotating electric field

spin-orbit interaction
preserves time-reversal
symmetry





spin-orbit interaction

* Electron moving in an electric field experiences a "magnetic" field in its rest-frame

$$\mathbf{B}_{\text{eff}} \sim \mathbf{E} \times \mathbf{p}/mc^2$$

$$\mathcal{H}_{\text{so}} \sim \mu_B \boldsymbol{\sigma} \cdot \mathbf{E} \times \mathbf{p}/mc^2$$

Semiconductors

* In solids the electric field is the gradient of the crystal potential \rightarrow "magnetic field" odd in the momentum to preserve time-reversal symmetry

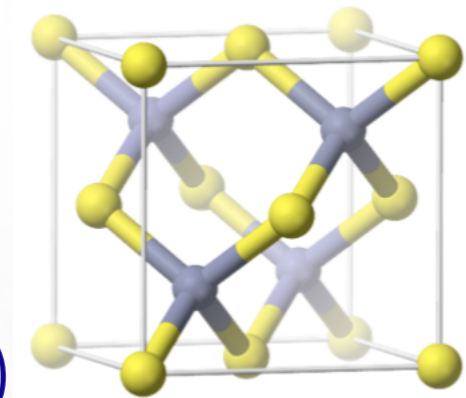
* In two-dimensions the interaction is integrated over the growth direction \rightarrow linear in the momentum

* bulk inversion asymmetry \rightarrow Dresselhaus

$$\mathcal{H}_D^{2d} = \beta(-p_x \sigma_x + p_y \sigma_y)$$

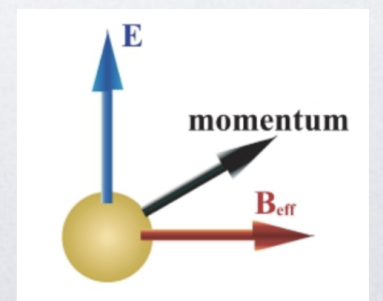
* structural inversion asymmetry \rightarrow Rashba

$$\mathcal{H}_R^{2d} = \alpha(p_x \sigma_y - p_y \sigma_x)$$



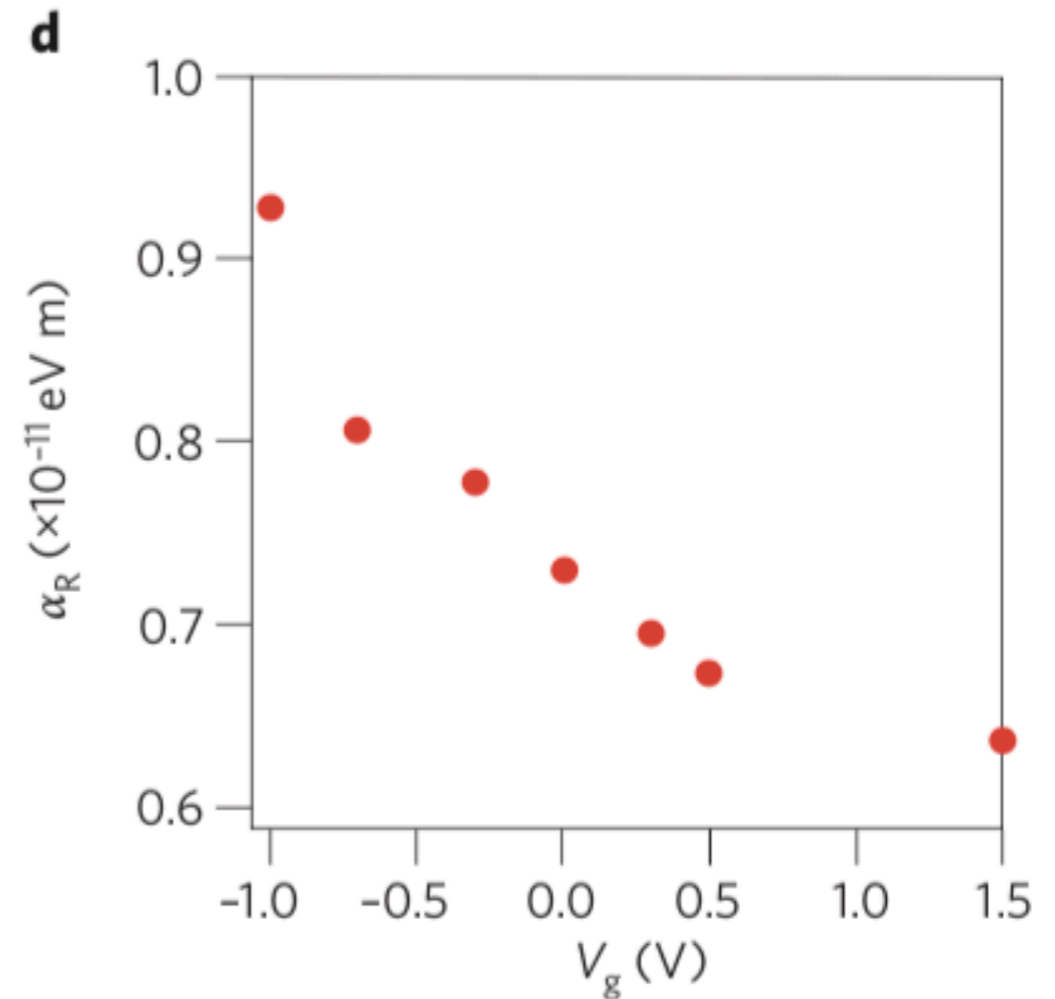
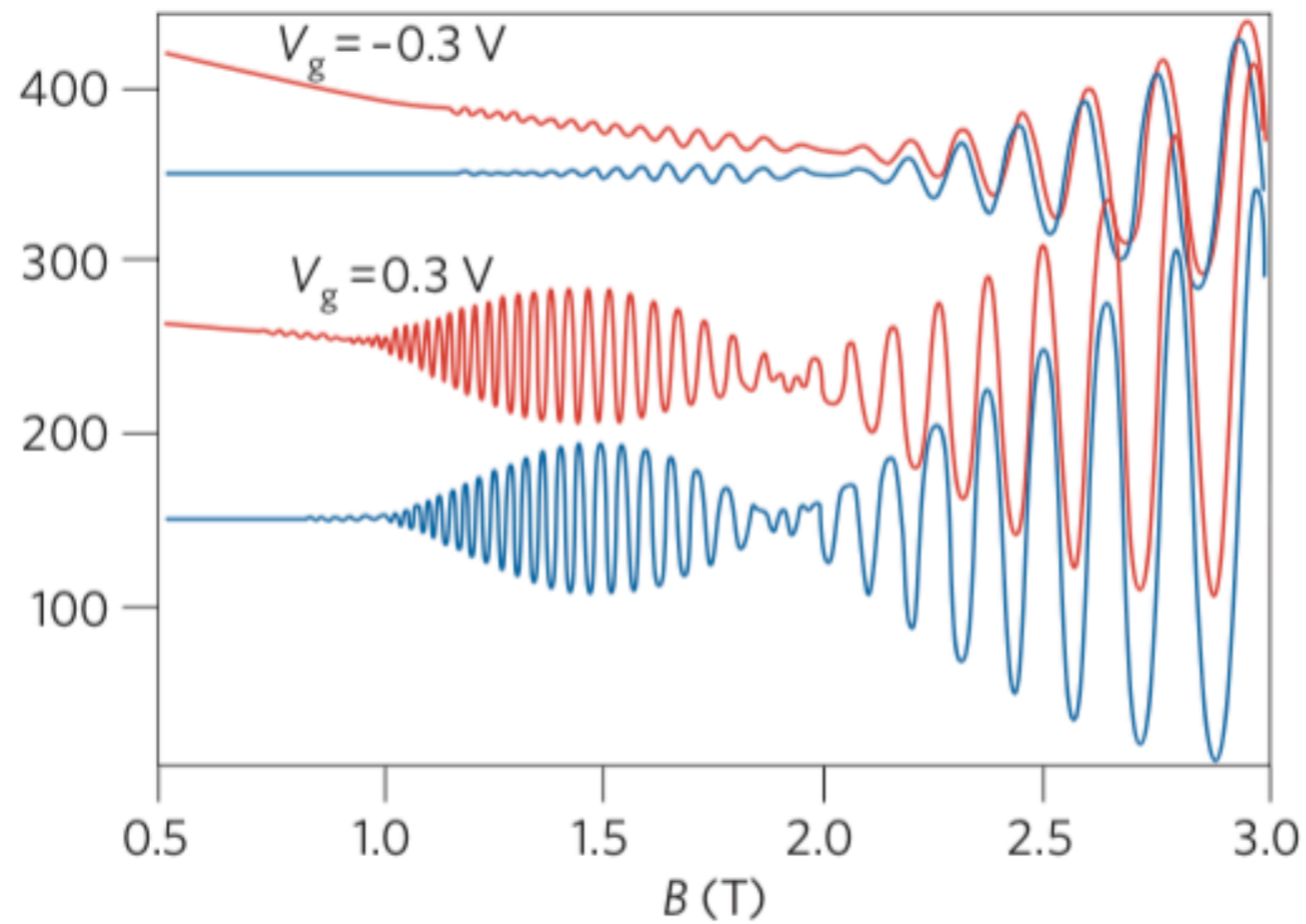
zinc-blende GaAs

GaAs/AlGaAs





Manipulating Rashba spin-orbit coupling



Nitta, Akazaki, Takayanagi, Enoki, Gate control of spin-orbit interaction in an inverted InGaAs/InAlAs heterostructures, PRL 78 (1997)



as the electron moves ballistically a distance L , the angle by which the spin is rotated by the linear spin-orbit interaction is independent of the velocity, i.e., the rotation angle is determined by the spin-orbit coupling and by L

experimental parameters

$l_{\text{so}} \Leftrightarrow \text{rotation by } \pi$

Dresselhaus spin-orbit parameter for GaAs $l_{\text{so}} = \hbar^2 / (\beta m^*) \sim 1 - 10 \mu\text{m}$

Dresselhaus spin-orbit parameter for dual-gated

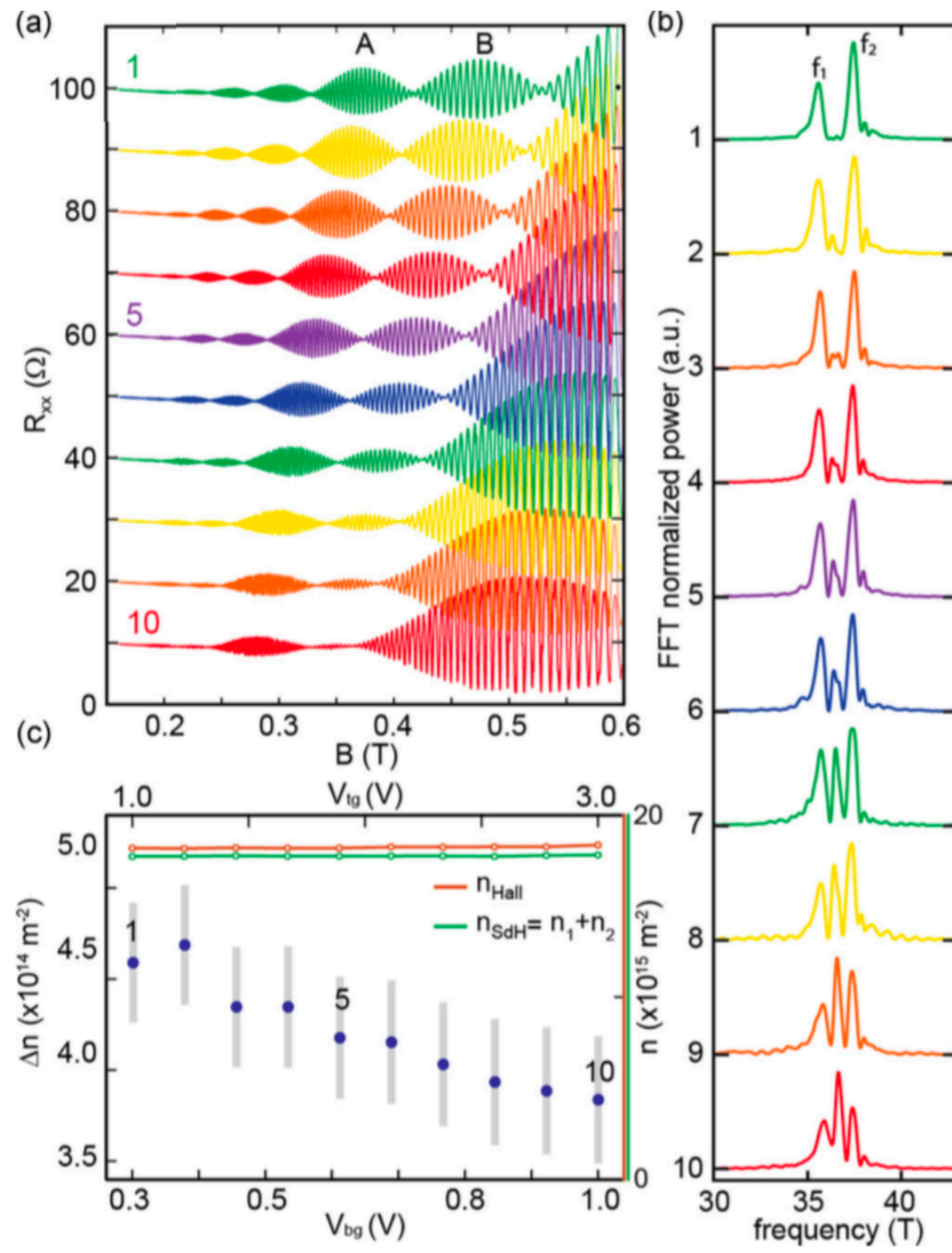
InAs/GaSb quantum well $l_{\text{so}} \sim .5 \mu\text{m}$ (28.5 meVÅ)

Rashba spin-orbit parameter for dual-gated

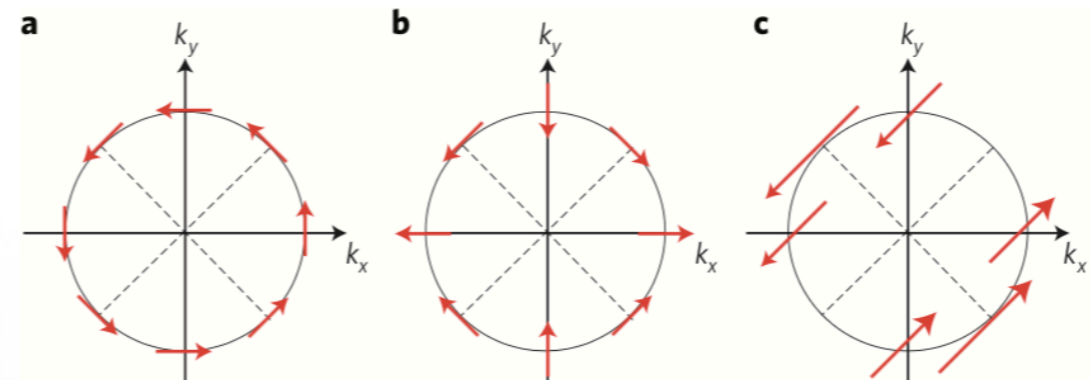
InAs/GaSb quantum well $l_{\text{so}} = \hbar^2 / (\alpha m^*) \sim .125 - .25 \mu\text{m}$ (75 ~ 53 meVÅ)

Rashba spin-orbit parameter for inversion layer of
the heterostructure $\text{In}_{0.75}\text{Ga}_{0.25}\text{As} / \text{In}_{0.75}\text{Al}_{0.25}\text{As}$

$l_{\text{so}} \sim .06 - .125 \mu\text{m}$



Rashba interaction and Dresselhaus interaction can add up—(110) direction



Manchon, Koo, Nitta, Frolov, Duine, Perspectives for Rashba spin-orbit coupling, Nat. Materials, 14 (2015)

Beukman, de Vries, van Veen, Skolasinski, Wimmer, Qu, de Vries, Nguyen, Yi, Kiselev, Sokolich, Manfra, Nichele, Marcus, Kouwenhoven, Spin-orbit interaction in dual gated InAs/GaSb quantum well, PRB 96 (2017)



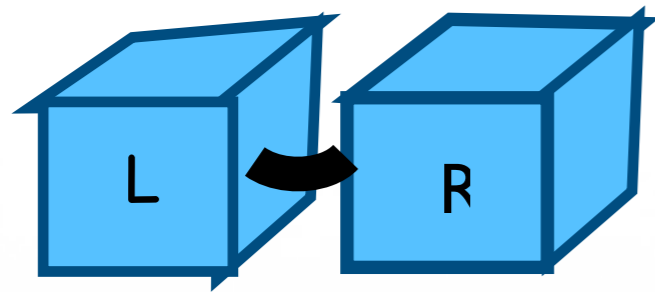
Large spin-orbit coupling in carbon nanotubes

G.A. Steele¹, F. Pei¹, E.A. Laird¹, J.M. Jol¹, H.B. Meerwaldt¹ & L.P. Kouwenhoven¹

It has recently been recognised that the strong spin-orbit interaction present in solids can lead to new phenomena, such as materials with non-trivial topological order. Although the atomic spin-orbit coupling in carbon is weak, the spin-orbit coupling in carbon nanotubes can be significant due to their curved surface. Previous works have reported spin-orbit couplings in reasonable agreement with theory, and this coupling strength has formed the basis of a large number of theoretical proposals. Here we report a spin-orbit coupling in three carbon nanotube devices that is an order of magnitude larger than previously measured. We find a zero-field spin splitting of up to 3.4 meV, corresponding to a built-in effective magnetic field of **29 T** aligned along the nanotube axis. Although the origin of the large spin-orbit coupling is not explained by existing theories, its strength is promising for applications of the spin-orbit interaction in carbon nanotubes devices.

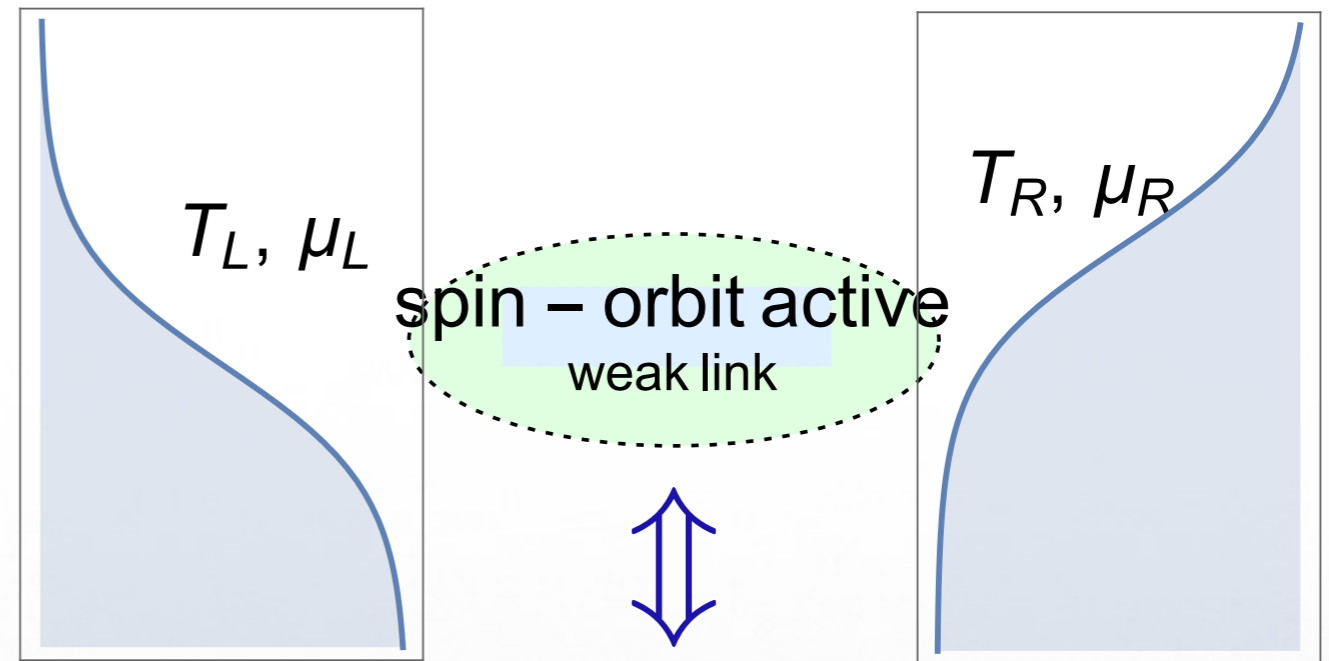


propagation through a "Rashba-effective" link



Bychkov and Rashba, Oscillatory effects and the magnetic susceptibility of carriers in inversion layers, J. Phys. C 17 (1984)

phenomenological Hamiltonian for spin-orbit coupling in uniaxial-symmetric systems lacking inversion symmetry (heterostructures, surfaces)



$$\mathcal{H}_R = \alpha_R (\mathbf{E} \times \mathbf{p}) \cdot \boldsymbol{\sigma}$$



Tunneling matrix element?

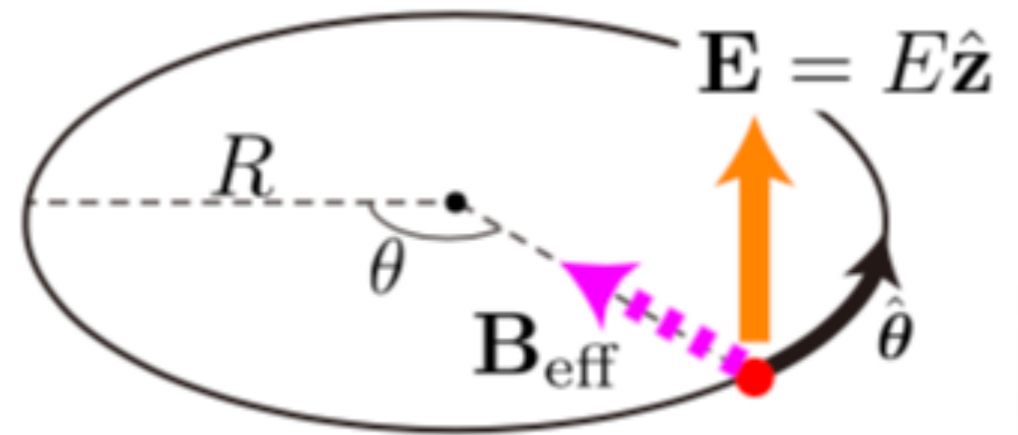


propagation through a "Rashba-effective" link

The Schrodinger equation for an electron moving on a ring, subjected to the Rashba interaction

$$\left[-i \frac{d}{d\theta} + k_{\text{so}} \hat{\mathbf{n}}(\theta) \cdot \boldsymbol{\sigma} \right]^2 \psi(\theta) = \epsilon \psi(\theta) ,$$

$$\hat{\mathbf{n}}(\theta) = [\cos(\theta), \sin(\theta), 0] \quad \hat{\mathbf{n}} \parallel \mathbf{B}_{\text{eff}}$$



$$\psi(\theta) = P(\theta) \tilde{\psi}(\theta) ,$$

Gauge transformation:

$$i \frac{d}{d\theta} P(\theta) = k_{\text{so}} \hat{\mathbf{n}}(\theta) \cdot \boldsymbol{\sigma}$$



propagation through a "Rashba-effective" link

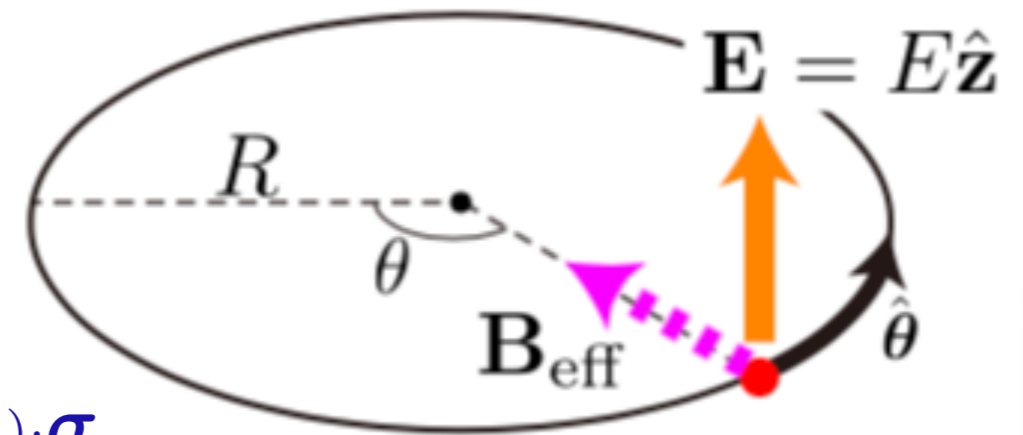
Electron moving through infinitesimal $d\theta$ acquires phase factor

$$\psi(\theta + d\theta) = \exp[ik_{so} \hat{\mathbf{n}}(\theta) \cdot \boldsymbol{\sigma} d\theta] \psi(\theta) ,$$

$$\hat{\mathbf{n}}(\theta) = \hat{\mathbf{x}} \cos(\theta) + \hat{\mathbf{y}} \sin(\theta)$$

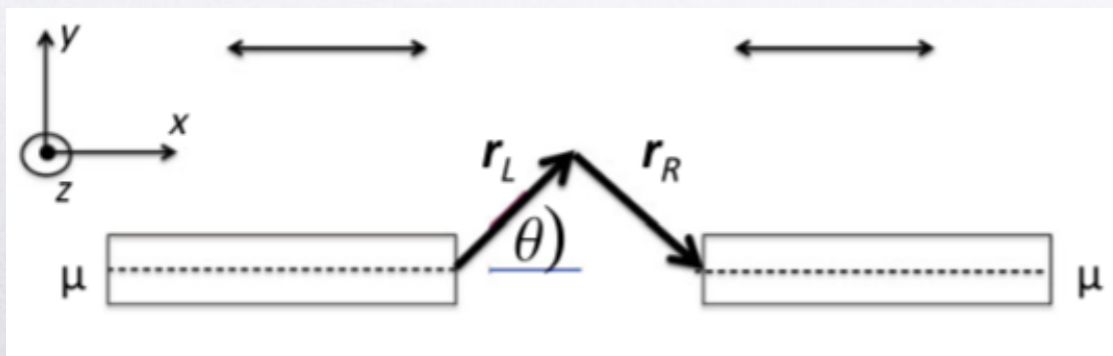
but: $e^{ik_{so} \hat{\mathbf{n}}(\theta_1) \cdot \boldsymbol{\sigma}} e^{ik_{so} \hat{\mathbf{n}}(\theta_2) \cdot \boldsymbol{\sigma}} \neq e^{ik_{so} \hat{\mathbf{n}}(\theta_2) \cdot \boldsymbol{\sigma}} e^{ik_{so} \hat{\mathbf{n}}(\theta_1) \cdot \boldsymbol{\sigma}}$

there appears effective magnetic field along $\hat{\mathbf{z}}$



$$\hat{\mathbf{n}} \parallel \mathbf{B}_{\text{eff}}$$

example:



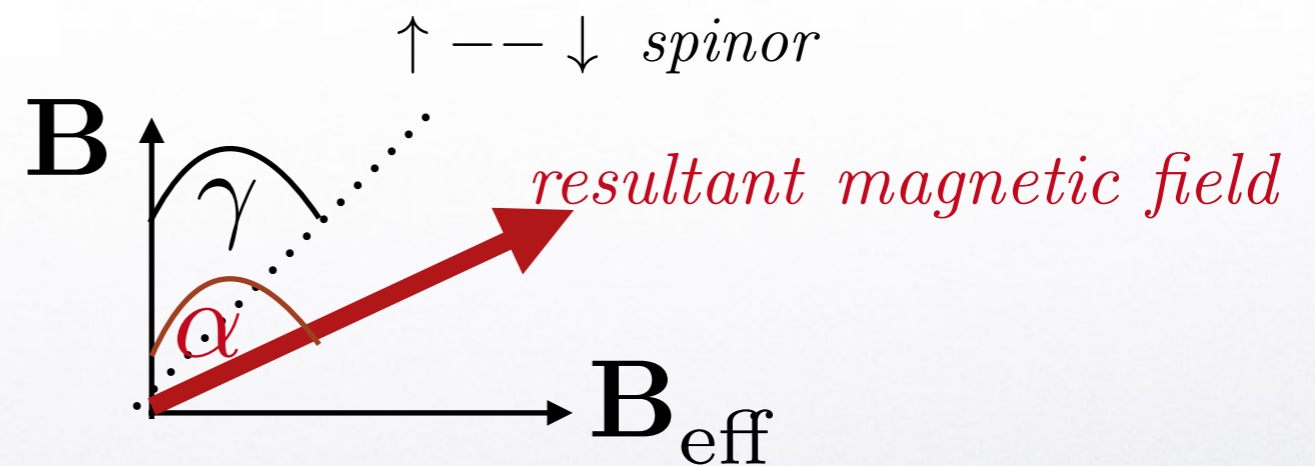
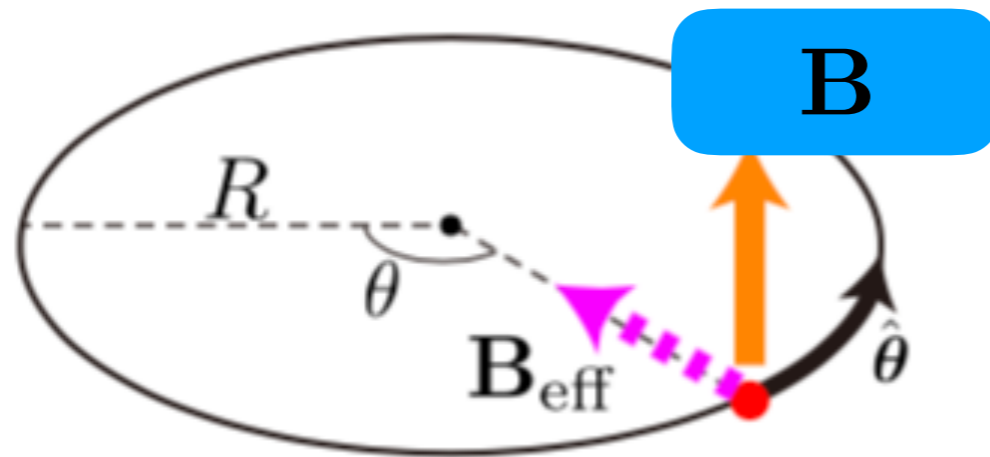
$$e^{-ik_{so} d \hat{\mathbf{n}}_L \cdot \boldsymbol{\sigma}} e^{-ik_{so} d \hat{\mathbf{n}}_R \cdot \boldsymbol{\sigma}}$$

$$= 1 - 2 \sin^2(k_{so} d) \cos^2(\theta)$$

$$+ i \sin(2k_{so} d) \cos(\theta) \sigma_y - i \sin^2(k_{so} d) \sin(2\theta) \sigma_z$$



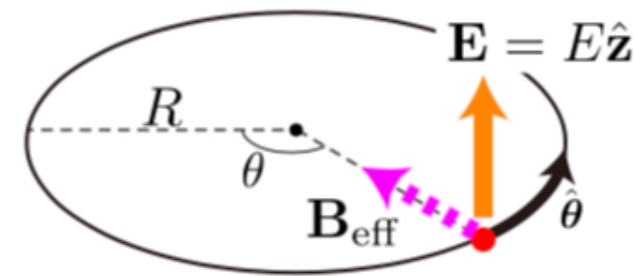
Berry phase—Ubiquitous explanation



$\gamma \Rightarrow \alpha$ in the adiabatic limit



propagation through a "Rashba-effective"
finite arc NO external magnetic field



$$\psi(\theta) = \begin{bmatrix} P_{11}(\theta) & P_{12}(\theta) \\ P_{21}(\theta) & P_{22}(\theta) \end{bmatrix} \psi(0)$$

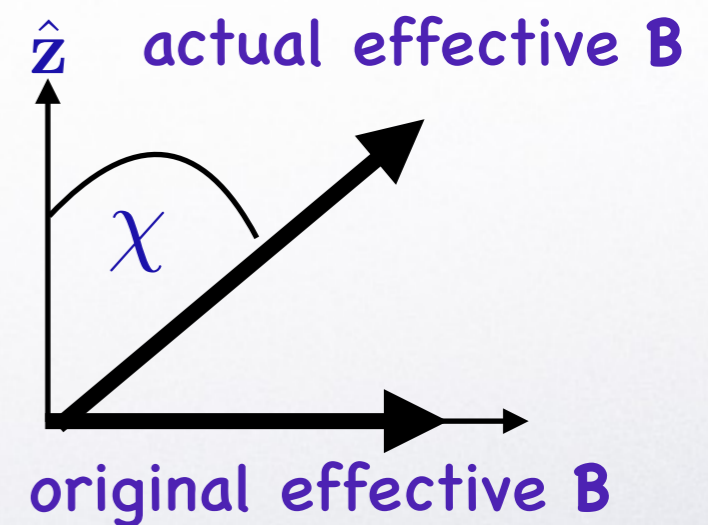
$$P_{11}(\theta) = P_{22}^*(\theta) = e^{-i\frac{\theta}{2}} \left(\cos\left[\left(a - \frac{1}{2}\right)\theta\right] - i \cos(\chi) \sin\left[\left(a - \frac{1}{2}\right)\theta\right] \right)$$

$$P_{12}(\theta) = P_{21}^*(\theta) = -ie^{-i\frac{\theta}{2}} \sin(\chi) \sin\left[\left(a - \frac{1}{2}\right)\theta\right]$$

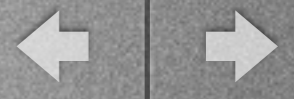
$$a = \frac{1 - \cos(\chi)}{2} + k_{\text{so}} \sin(\chi) \quad [\text{dimensionless } k_{\text{so}}]$$

Aharonov-Anandan
(Berry) phase

Dynamical phase



$$\tan(\chi) = -2k_{\text{so}} \Rightarrow a = [1 - 1/\cos(\chi)]/2$$



*Aharonov and Casher, Topological quantum effects for neutral particles, PRL 53, (1984)

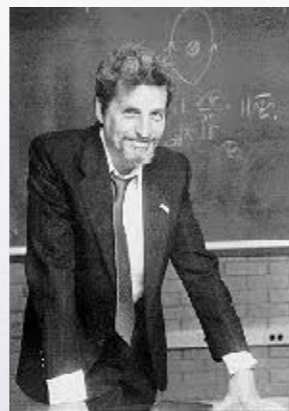
*Aharonov and Anandan, Phase change during a cyclic quantum evolution, PRL 58, (1987)

*Qian and Su, Spin-orbit interaction and Aharonov-Anandan phase in mesoscopic rings, PRL 72, (1994)

* Avishai, Totsuka, and Nagaosa, Non-abelian Aharonov-Casher phase factor in mesoscopic systems, arXiv: 1904.01751



A. Casher



Y. Aharonov



J. Anandan



tunneling amplitude—
the propagator

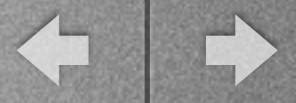
Shabhazyan and Raikh, Low-field anomaly in 2D hopping magnetoresistance caused by spin-orbit term in the energy spectrum, PRL 73, (1994)

propagation of a plane wave
(wave vector k) along a
straight segment of length s

$$P(E) = \int dk e^{iks} [E_F - \mathcal{H}(k)]^{-1}$$

$$\mathcal{H} = \frac{1}{2m^*} \left(-i \frac{d}{ds} - \frac{e}{c} \mathbf{A} \right)^2 + \frac{\tilde{k}_{so}}{m^*} \hat{\mathbf{n}} \cdot \boldsymbol{\sigma} \times \left(-i \frac{d}{ds} - \frac{e}{c} \mathbf{A} \right) - \mathbf{B} \cdot \boldsymbol{\sigma} \quad \Rightarrow \quad \mathcal{H}(k) = \frac{k^2}{2m^*} + \frac{k k_{so}}{m^*} (\hat{\mathbf{n}} \times \hat{\mathbf{s}}) \cdot \boldsymbol{\sigma}$$

omit Aharonov-Bohm phase due to \mathbf{A} , ignore Zeeman interaction due to \mathbf{B} , assume $\mathbf{n} \parallel \mathbf{E}$ normal to plane where \mathbf{k} is



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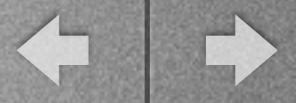
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propagation of a plane wave
(wave vector k) along a
straight segment of length s

propagation amplitude is a unitary
matrix in the Hilbert space of the spin

Tunneling amplitude
also a unitary matrix

$$P(E) = -\pi m^* \frac{e^{-s\sqrt{(1/a^2) - k_{so}^2}}}{\sqrt{(1/a^2) - k_{so}^2}} \exp[ik_{so}\hat{n} \times \hat{s} \cdot \sigma]$$

time-reversal symmetry \rightarrow no
spin polarization in two-terminal
junctions

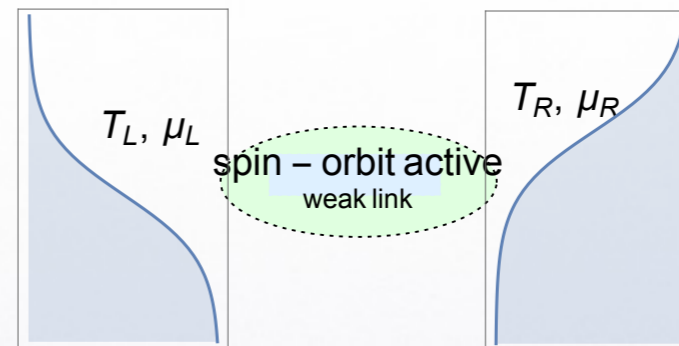
$$P(E) = \int dk e^{iks} [E_F - \mathcal{H}(k)]^{-1}$$

Cauchy integration



$$= -i\pi m^* \frac{e^{is\sqrt{k_F^2 + k_{so}^2}}}{\sqrt{k_F^2 + k_{so}^2}} \exp[ik_{so}\hat{n} \times \hat{s} \cdot \sigma]$$

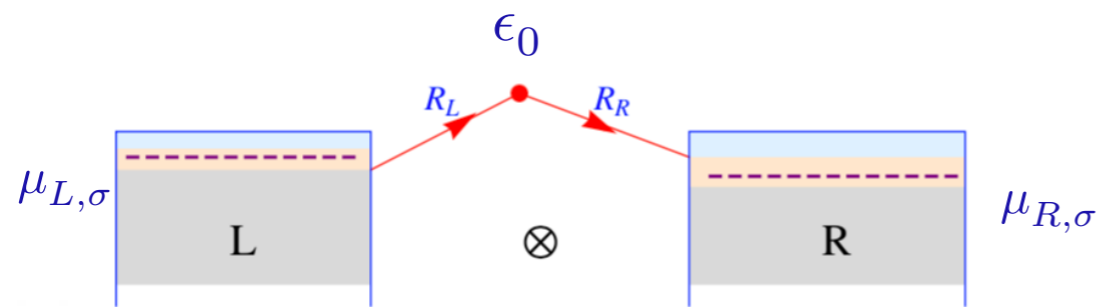
$$E_F < \mu$$



Bardarson, a proof of Kramers degeneracy
of transmission eigenvalues from antisymmetry
of the scattering matrix, J. Phys. A 41 (2008)



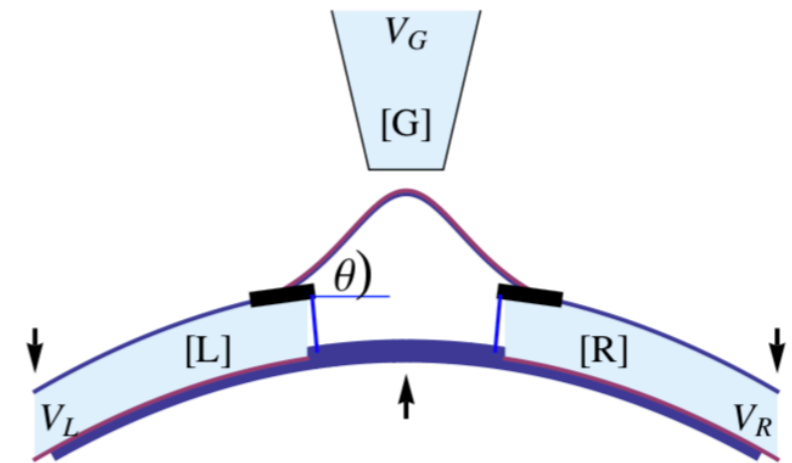
Mechanically controlled Rashba spin splitter



The model exploited in the calculations: the location of the quantum dot vibrates normal to the wire in the junction plane

$$I_{L,\sigma} = \frac{\Gamma_L \Gamma_R}{2\pi \epsilon_0^2} \sum_{\sigma'} \sum_{n,n'=0}^{\infty} P(n) |\langle n | [e^{-i\psi_R} e^{-i\psi_L}]_{\sigma',\sigma} | n' \rangle|^2 \times (1 - e^{\beta(\mu_{L,\sigma} - \mu_{R,\sigma'})}) \frac{\mu_{L,\sigma} - \mu_{R,\sigma'} + (n' - n)\omega}{e^{\beta[\mu_{L,\sigma} - \mu_{R,\sigma'} + (n' - n)\omega]} - 1}$$

$$P(n) = (1 - e^{-\beta\omega}) e^{-n\beta\omega}$$



Shekhter, OEW, and Aharony, Suspended nanowires as mechanically-controlled Rashba spin filters, PRL 111 (2013)

The spin-orbit interaction in the bent wire can be modulated mechanically by loads and electrically, by biasing the STM



Shekhter, Gorelik, Glazman, and Jonson, Electronic Aharonov-Bohm effect induced by quantum vibrations, PRL 97 (2006)

Electrons tunneling through the weak link (e.g., SWNT) excite flexural vibrations in the presence of Aharonov-Bohm type magnetic field

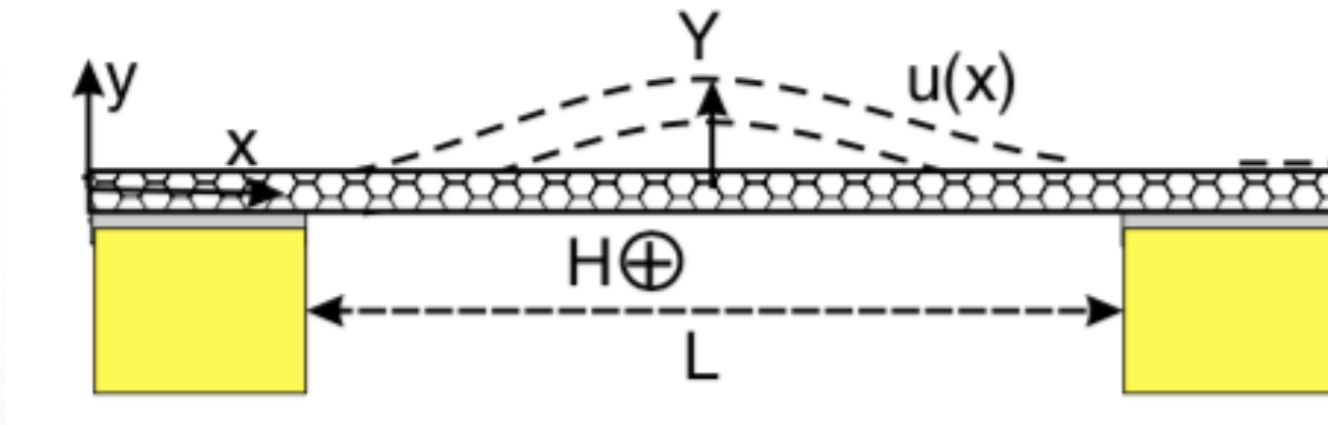


FIG. 2 (color online). Nanoelectromechanical system proposed to show the coherent coupling between quantum electron transport and quantum flexural vibrations discussed in the text. Electrons tunneling through a doubly clamped SWNT excite quantized vibrations of the SWNT in the presence of a magnetic field, H . The resulting effective multiconnectivity of the system leads to a negative magnetoconductance (see text).



Unpolarized leads

$$\mu_{L(R),\sigma} = \mu_{L(R)}$$

$$I_{L,\sigma} = \frac{\Gamma_L \Gamma_R}{2\pi\epsilon_0^2} \sum_{\sigma'} \sum_{n,n'=0}^{\infty} P(n) |\langle n | [e^{-i\psi_R} e^{-i\psi_L}]_{\sigma'\sigma} | n' \rangle|^2 \\ \times (1 - e^{\beta(\mu_{L,\sigma} - \mu_{R,\sigma'})}) \frac{\mu_{L,\sigma} - \mu_{R,\sigma'} + (n' - n)\omega}{e^{\beta[\mu_{L,\sigma} - \mu_{R,\sigma'} + (n' - n)\omega]} - 1}$$

$$P(n) = (1 - e^{-\beta\omega}) e^{-n\beta\omega}$$

Summing over spin indices (to obtain the charge current) implies

$$\delta_{n,n'}$$

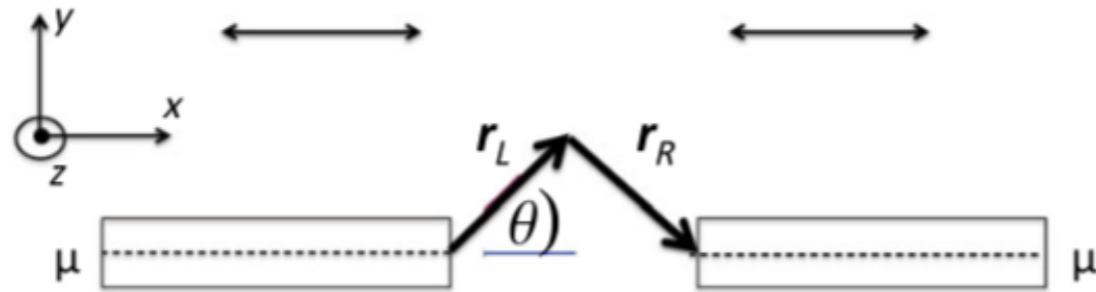
leading to the Landauer formula (particle current)

(almost) no effect of Rashba interaction on the tunneling conductance

$$I = (\mu_R - \mu_L) \frac{\Gamma_L \Gamma_R}{\pi\epsilon_0^2}$$



Details of the spin-orbit interaction



$$e^{-i\psi_L} e^{-i\psi_R}$$

$$\begin{aligned} & e^{-ik_{so}d\hat{\mathbf{n}}_L \cdot \boldsymbol{\sigma}} e^{-ik_{so}d\hat{\mathbf{n}}_R \cdot \boldsymbol{\sigma}} \\ &= 1 - 2\sin^2(k_{so}d)\cos^2(\theta) \\ &+ i\sin(2k_{so}d)\cos(\theta)\sigma_y - i\sin^2(k_{so}d)\sin(2\theta)\sigma_z \end{aligned}$$

Rashba scatterer as a spin source

$$\theta = \theta_0 + \frac{a_0 \cos(\theta_0)}{d} (b + b^\dagger)$$

$$\begin{aligned} \mu_{L(R)\sigma} &= \mu + \sigma \frac{U_{L(R)}}{2} \\ U &= \frac{U_L + U_R}{2} \end{aligned}$$

$$I_{L,\sigma} + I_{R,\sigma} = U \frac{\Gamma_L \Gamma_R}{\pi \epsilon_0^2} \sin^2(2k_{so}d)$$

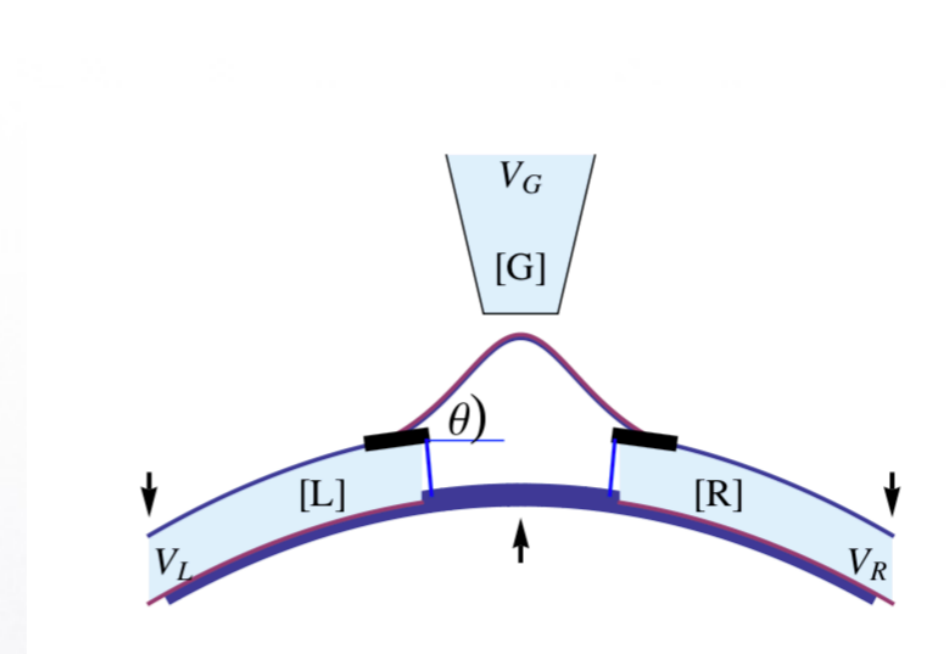
For a certain spin component:

$$\times \sum_{n=0}^{\infty} \sum_{\ell=1}^{\infty} P(n) |\langle n | \cos(\theta) | n + \ell \rangle|^2 \frac{2\beta \ell \omega}{e^{\beta \ell \omega} - 1}$$



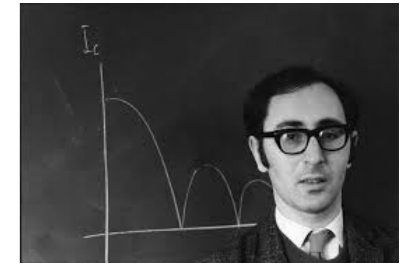
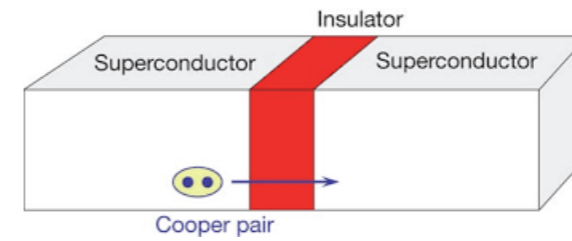
Rashba spin splitter

Dynamic of the spin is deterministic (i.e., not random as from magnetic impurities) \Rightarrow interference of spins in nanostructures with spatially localized spin-orbit interaction induces spin currents in unpolarized conductors, currents which are not associated with charge transportation. The Rashba spin splitter can be designed by mechanically tuning the nanowire.





Rashba splitting of Cooper pairs Josephson current



Splitting of the spin state of paired electrons (that carry the Josephson current)

⇒ interference between the channel where $\uparrow\downarrow$ and the channel where $\uparrow\uparrow$



interference pattern in the amplitude of the Josephson current (not there in the normal conductance)

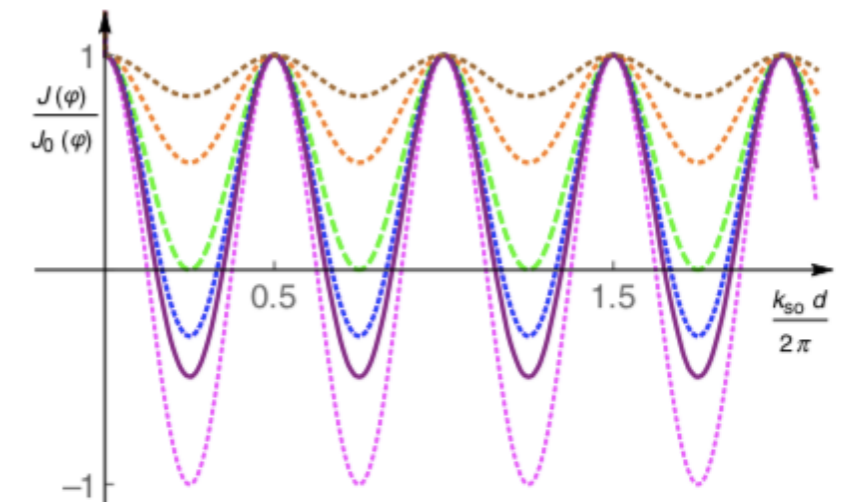


FIG. 3. The Josephson current $J(\varphi)$ divided by its value without the SO interaction, $J_0(\varphi)$, for the genuine Rashba configuration [Eqs. (7) and (8)] as a function of $k_{so}d/(2\pi)$. The largest amplitude is for the zero bending angle, $\theta = 0$, decreasing gradually for $\theta = \pi/6, \pi/5, \pi/4, \pi/3, \pi/2.5$ [Fig. 2(b)]. Relevant values of k_{so} are estimated in the text.



Josephson equilibrium current

- *electrons in the source are paired in time-reversed states in which their spins are antiparallel
- *supercurrent carried by Cooper pairs flows in the non-superconducting link when there is a (order-parameter) phase difference across the weak link
- *the electrons tunnel one-by-one but the Cooper pairs maintain superconducting coherence (link shorter than the coherence length)
- *when reaching the drain the electrons are paired again



Josephson equilibrium current

*Supercurrent is proportional to the superconducting phase difference

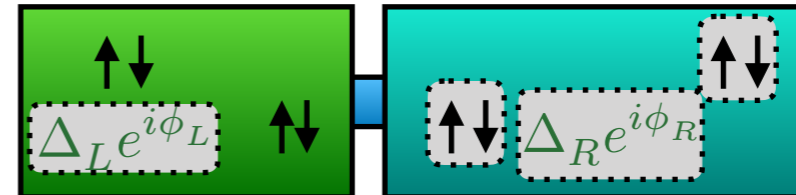
$$I(\varphi) = I_0 \sin(\varphi)$$

$$\varphi = \phi_L - \phi_R$$

*the critical current for a junction with identical superconductors

$$I_0 = \frac{\pi \Delta}{2e} G_n$$

* G_n is the normal-state conductance



Ambegaokar and Baratoff,
tunneling between
superconductors, PRL 10, (1963)

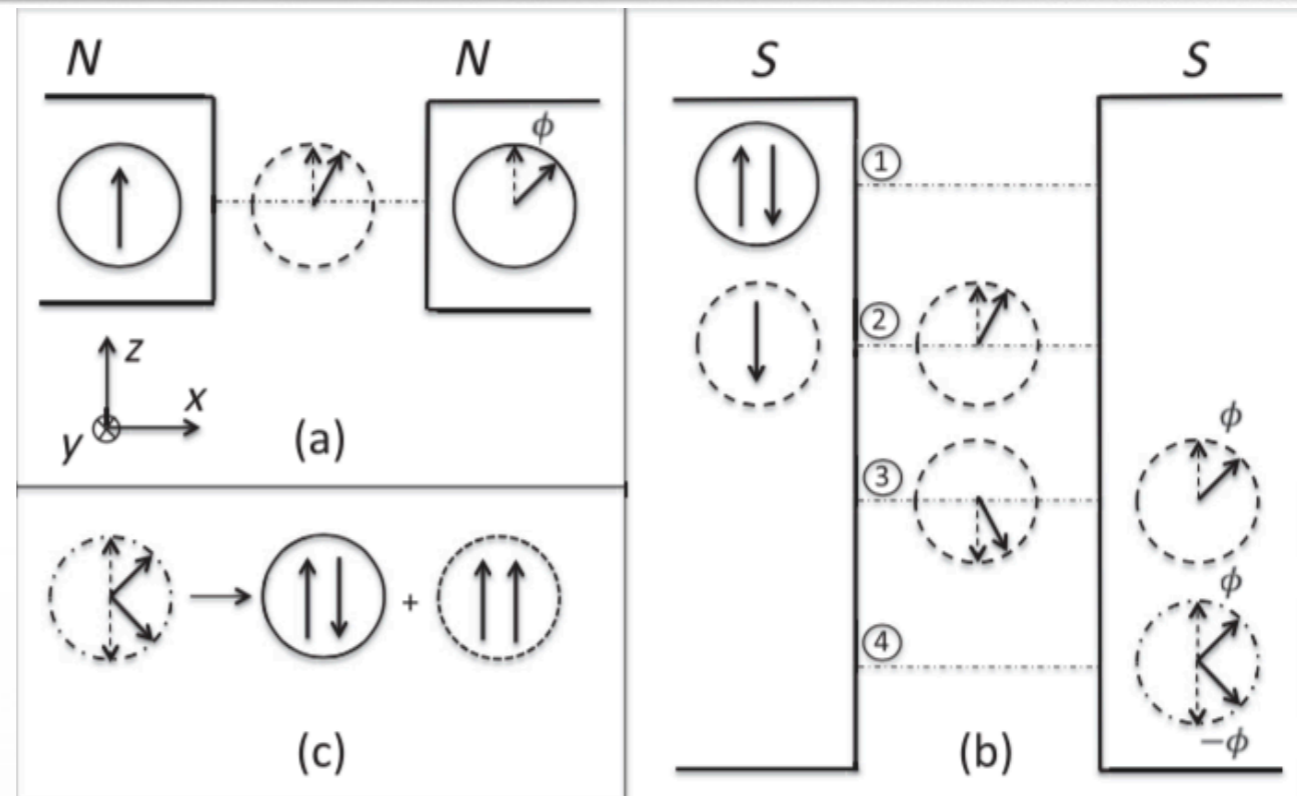


Illustration (semi-classical description)

$$\phi \propto \text{spin - orbit coupling } (k_{so}d)$$

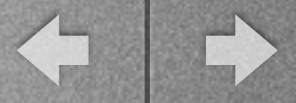
Evolution of spin states of electrons moving between two bulk leads via link in which they are subjected to Rashba interaction (effective magnetic field directed along y)

N-N: if the spin in left is along z , the spin at right is in a coherent superposition of \uparrow, \downarrow



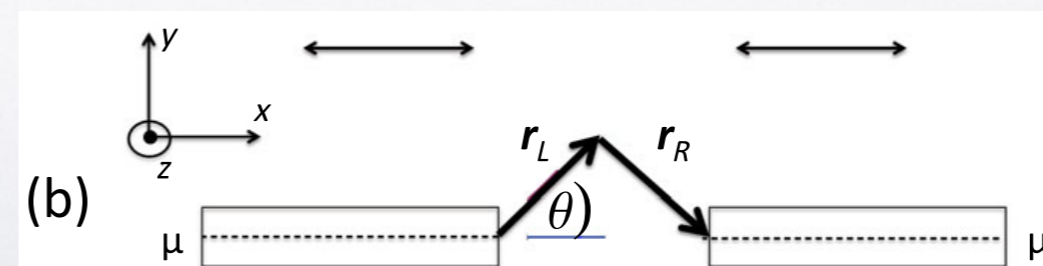
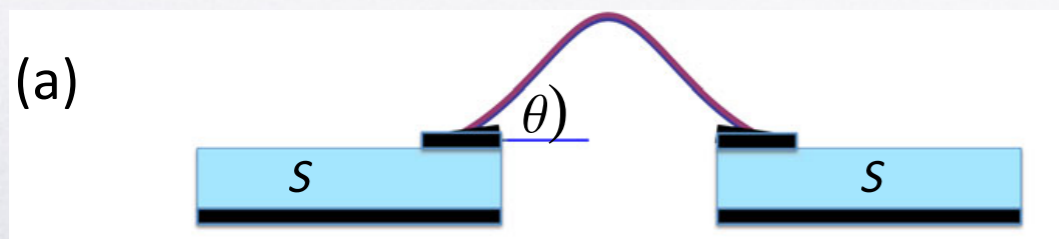
sequential transfer

Cooper pair S-S (Coulomb-blockade-no double occupancy on the link) two electrons in time-reversed quantum states, their spins are reversed wrt one another, rotation angles have opposite signs. Final state is a coherent mixture of spin-singlet and spin-triplet state, but only the former can enter into the second S, leading to a reduction in the amplitude of the Josephson current.



Transfer of Cooper pairs through a weak link

- *Cooper pairs are transferred between superconducting leads via virtual localized states
- *Coulomb blockade—the members of the pair are transferred sequentially
- *short weak link—dependence of the matrix element for a single-electron transfer on electron energy in the virtual state can be ignored $d < \hbar v_F / |\Delta|$
- *conservation of longitudinal momentum

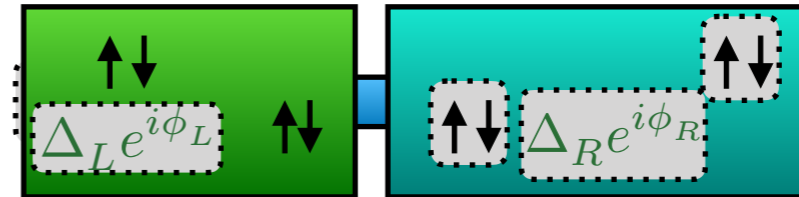




Josephson tunneling

$$\mathcal{H} = \mathcal{H}_L + \mathcal{H}_R + \mathcal{H}_T$$

$$\mathcal{H}_L = \sum_{\mathbf{k}} \sum_{\sigma} (\epsilon_{\mathbf{k}} - \mu) c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma} + \left(\Delta_L e^{i\phi_L} \sum_{\mathbf{k}} c_{\mathbf{k}\uparrow}^{\dagger} c_{-\mathbf{k}\downarrow}^{\dagger} + \text{H.c.} \right)$$

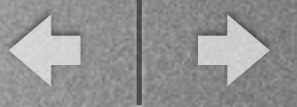


Single-electron tunneling

$$\mathcal{H}_T = \sum_{\mathbf{k}, \mathbf{p}} \sum_{\sigma, \sigma'} \left(c_{\mathbf{p}\sigma'}^{\dagger} [W_{\mathbf{p}\mathbf{k}}]_{\sigma'\sigma} c_{\mathbf{k}\sigma} + \text{H.c.} \right)$$

Time-reversal symmetry

$$[W_{\mathbf{p}\mathbf{k}}]_{\sigma\sigma'} = ([W_{-\mathbf{p}-\mathbf{k}}]_{-\sigma-\sigma'})^*$$



Josephson current

For $W_{\mathbf{k}\mathbf{p}} = W_0 \mathcal{W}$

calculation of the current
(second-order in the tunneling)

$$I_L = -e \langle \dot{N}_L \rangle = -ie \langle [\mathcal{H}, \sum_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma}] \rangle$$

gives for the equilibrium (no bias) Josephson current

$$\frac{I(\varphi)}{I_0(\varphi)} = \frac{1}{2} \sum_{\sigma} ([\mathcal{W}]_{\sigma\sigma} - [\mathcal{W}]_{\sigma-\sigma})$$

Without the spin-orbit interaction $\mathcal{W}_{\sigma\sigma}$ is diagonal in spin space

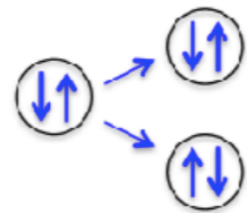


Rashba splitting of Cooper pairs (in the Coulomb blockade regime)

equilibrium (no bias) Josephson current

$$\frac{I(\varphi)}{I_0(\varphi)} = \frac{1}{2} \sum_{\sigma} [|\mathcal{W}_{\sigma\sigma}|^2 - |\mathcal{W}_{\sigma\bar{\sigma}}|^2] = 1 - 2 \cos^2(\theta) \sin^2(k_{\text{so}}d)$$

Josephson tunneling π shift



special cases :

$$\theta = 0, \quad k_{\text{so}}d = \pi/4, \pi/2$$

Normal-state conductance is unchanged

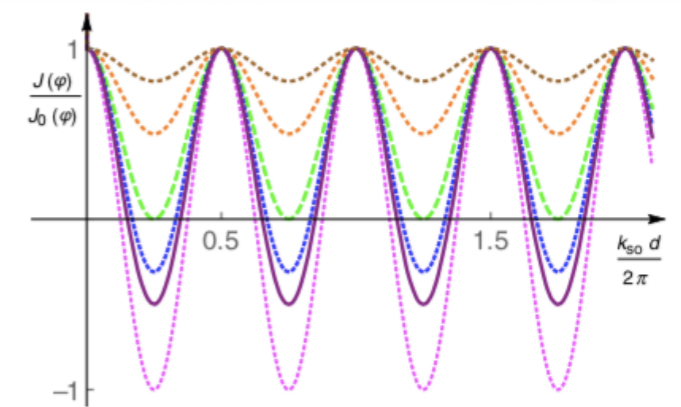
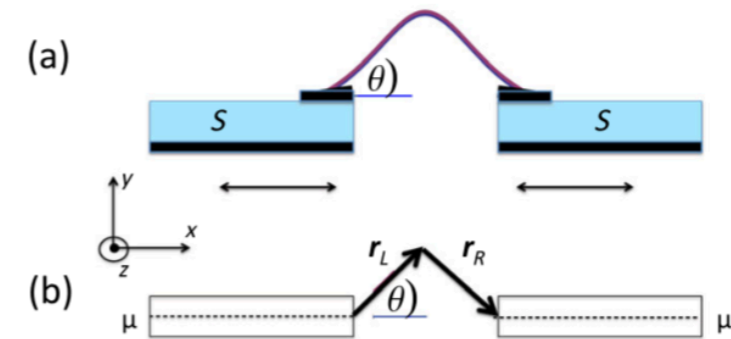
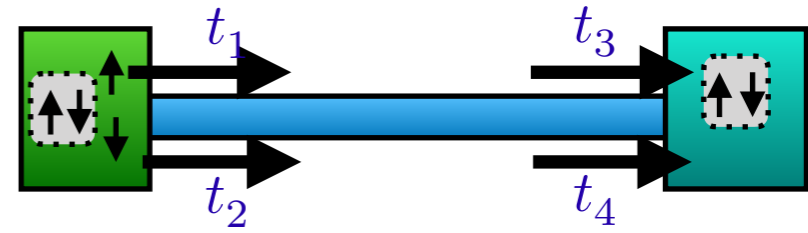


FIG. 3. The Josephson current $J(\varphi)$ divided by its value without the SO interaction, $J_0(\varphi)$, for the genuine Rashba configuration [Eqs. (7) and (8)] as a function of $k_{\text{so}}d/(2\pi)$. The largest amplitude is for the zero bending angle, $\theta = 0$, decreasing gradually for $\theta = \pi/6, \pi/5, \pi/4, \pi/3, \pi/2.5$ [Fig. 2(b)]. Relevant values of k_{so} are estimated in the text.



Lifting the Coulomb blockade

(will this destroy the coherent spin precession)



1st electron comes into the link at t_1 leaves at t_3

2nd electron comes into the link at t_2 leaves at t_4

“single-electron tunneling channel”

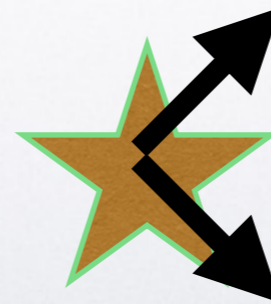
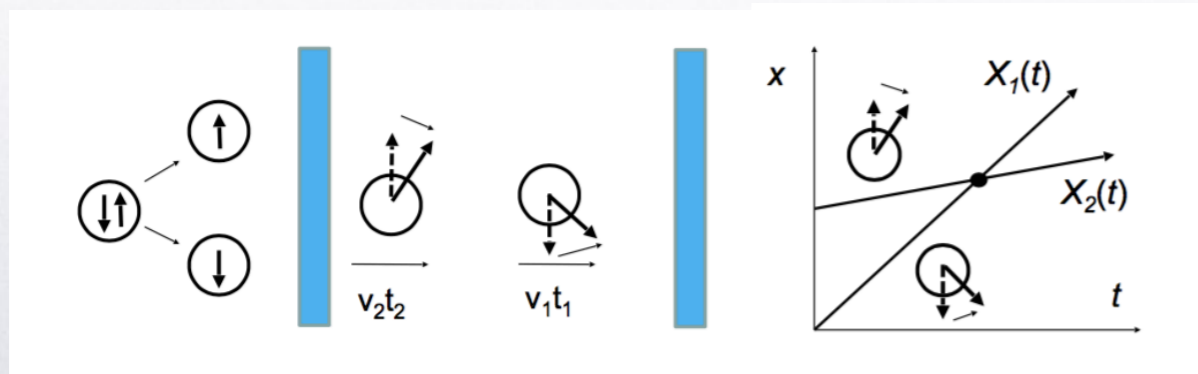
$$t_1 < t_3 < t_2 < t_4$$

electrons tunnel sequentially one by one

“double-electron tunneling channel”

$$t_1 < t_2 < t_3 < t_4$$

Coulomb interaction comes into play

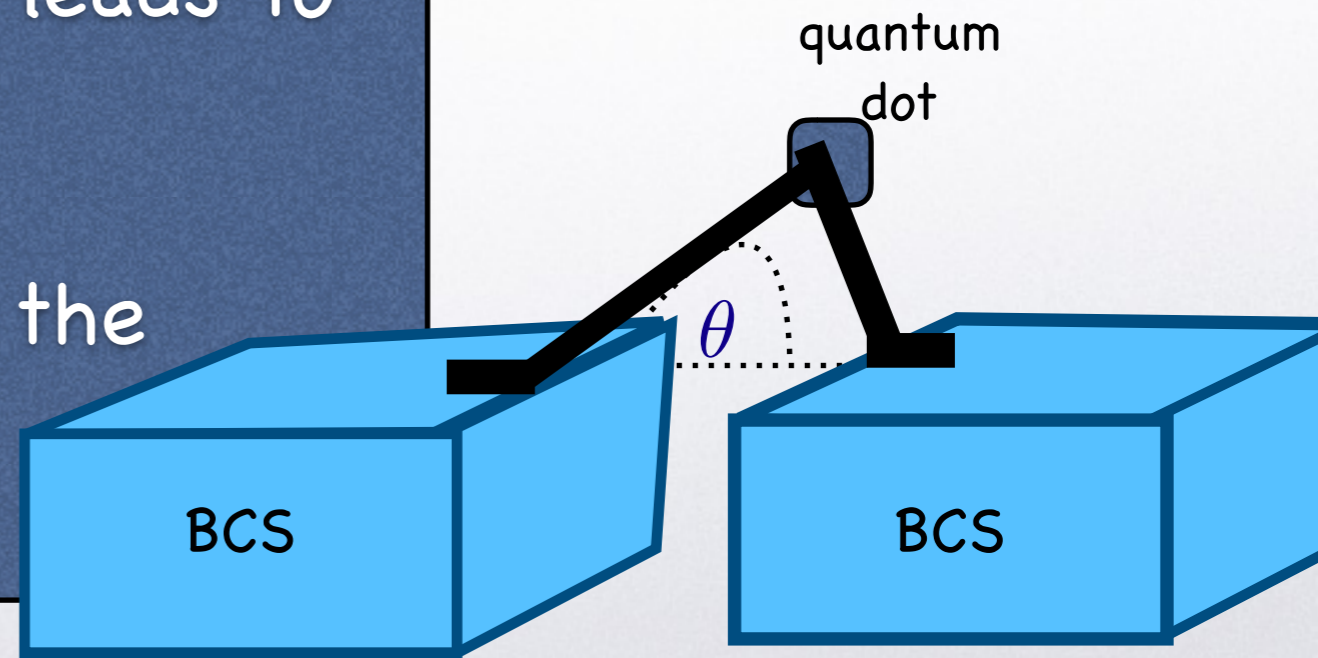


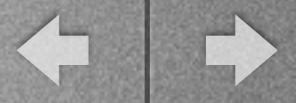
conflict with
Pauli principle



modeling spin precession of Cooper pairs

- *the "meeting point" is modeled by a quantum dot with spin-up and spin-down states; injection into these states obeys the Pauli principle (breaking the coherent evolution of spin states before and after the meeting)
- *single-electron tunneling from leads to the quantum dot
- *Coulomb repulsion on the dot
- *the results are averaged over the location of the quantum dot





System's Hamiltonian

Tunneling through dot:

$$\mathcal{H}_{\text{tun}} = \mathcal{H}_{LD} + \mathcal{H}_{RD} + \mathcal{H}_{DL} + \mathcal{H}_{DR}$$

$$\mathcal{H}_{LD} = \sum_{\mathbf{k}, \sigma, \sigma'} [t_{\mathbf{k}}]_{\sigma\sigma'} c_{\mathbf{k}\sigma}^\dagger d_{\sigma'}$$

1. Going from the time-reversed state of

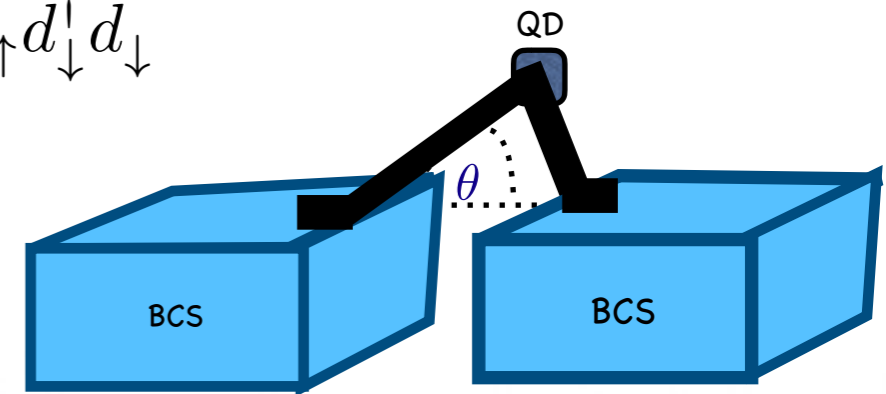
$$\mathbf{T} = K(i\sigma_y)$$

σ' , *i.e.*, $|\bar{\sigma}'\rangle = (i\sigma_y)|\sigma'\rangle$ to state $|- \mathbf{k}, \bar{\sigma}\rangle$

requires $\bar{\mathbf{t}}_{\mathbf{k}} = \mathbf{T}\mathbf{t}_{\mathbf{k}}\mathbf{T}^{-1}$

$$[\bar{\mathbf{t}}_{\mathbf{k}}]_{\bar{\sigma}\bar{\sigma}'} = [\mathbf{t}_{\mathbf{k}}^*]_{\sigma\sigma'}$$

$$\begin{aligned} \mathcal{H} &= \mathcal{H}_L + \mathcal{H}_R + \mathcal{H}_{\text{tun}} \\ &+ \sum_{\sigma} \epsilon d_{\sigma}^\dagger d_{\sigma} + U d_{\uparrow}^\dagger d_{\uparrow} d_{\downarrow}^\dagger d_{\downarrow} \end{aligned}$$



Shekhter, OEW, Jonson, Aharony, Spin precession in spin-orbit coupled weak links: Coulomb repulsion and Pauli quenching, PRB(R), 96 (2017)



$$I_L = -e \langle \dot{N}_L \rangle = -ie \langle [\mathcal{H}, \sum_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma}] \rangle$$

2. Terms in the perturbation theory

One particle on the link

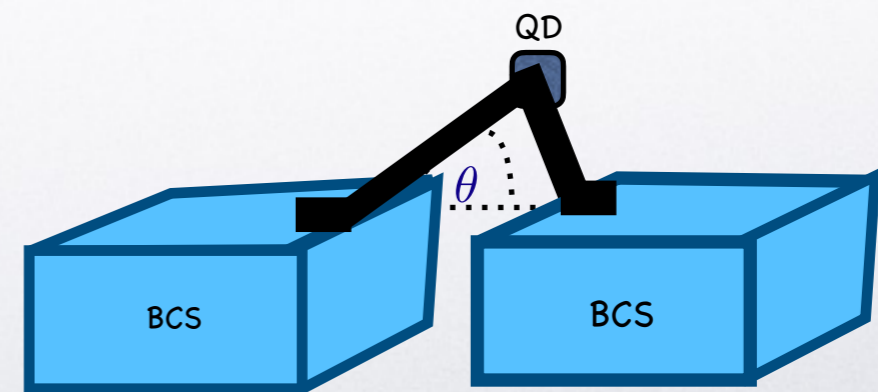
$$\langle \mathcal{H}_{LD}(t) \mathcal{H}_{DR}(t_1) \mathcal{H}_{LD}(t_2) \mathcal{H}_{DR}(t_3) \rangle$$
$$[t \geq t_1 \geq t_2 \geq t_3]$$

two particles on the link

$$\langle \mathcal{H}_{LD}(t) \mathcal{H}_{LD}(t_1) \mathcal{H}_{DR}(t_2) \mathcal{H}_{DR}(t_3) \rangle$$

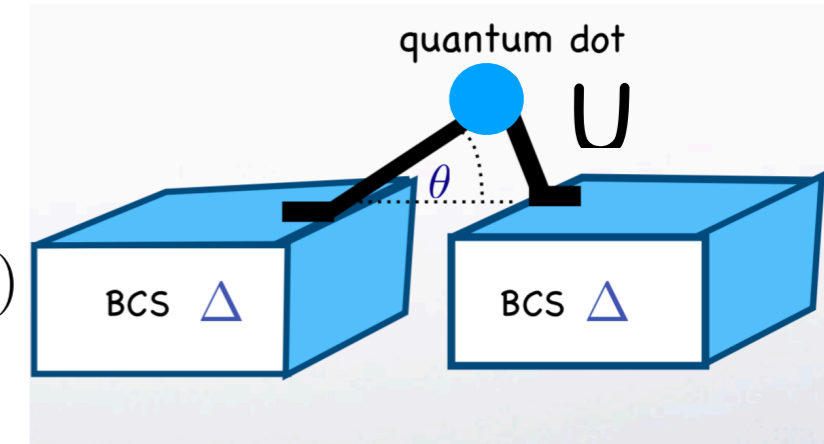
3. In the first scenario the tunneling amplitudes can be grouped to yield an effective L \leftrightarrow R amplitude, V_{LR} ;

In the second they cannot.





4. Spin dependent transmission



One particle on the link

$$\mathcal{T}_1 = \frac{1}{2} \sum_{\sigma} (|V_{LR} V_{LR}^{\dagger}]_{\sigma\sigma} - [V_{LR} V_{LR}^{\dagger}]_{\sigma-\sigma})$$

two particles on the link

$$\mathcal{T}_2 = \left(|[V_{DR}]_{\uparrow\uparrow}|^2 - |[V_{DR}]_{\uparrow\downarrow}|^2 \right) \left(|[V_{LD}]_{\uparrow\uparrow}|^2 - |[V_{LD}]_{\uparrow\downarrow}|^2 \right)$$

Loss of coherence at the quantum dot due to the Pauli principle

5. "energy denominator" due to quantum dot

$$F_1\left(\frac{\epsilon}{\Delta}\right) = \int_{-\infty}^{\infty} \frac{dx_1 dx_2}{\pi^2} \left[[\cosh(x_1) + \frac{\epsilon}{\Delta}] [\cosh(x_1) + \cosh(x_2)] [\cosh(x_2) + \frac{\epsilon}{\Delta}] \right]^{-1}$$

$$F_2\left(\frac{\epsilon}{\Delta}, \frac{U}{\Delta}\right) = \int_{-\infty}^{\infty} \frac{dx_1 dx_2}{\pi^2} \left[[\cosh(x_1) + \frac{\epsilon}{\Delta}] \left[2\frac{\epsilon}{\Delta} + \frac{U}{\Delta} \right] [\cosh(x_2) + \frac{\epsilon}{\Delta}] \right]^{-1}$$

Glazman and Matveev, Resonant Josephson current through Kondo impurities in a tunnel barrier, JEPT Lett., 49 (1989)



Breaking time-reversal symmetry by time-dependent electric fields

Spin-orbit interaction created by slowly-rotating electric field perpendicularly to the (one-dimensional) weak link



Even though the leads are non magnetic, and there is no bias, the rotating electric field creates DC spin current and transverse spin components which rotate around the link, flowing into the leads

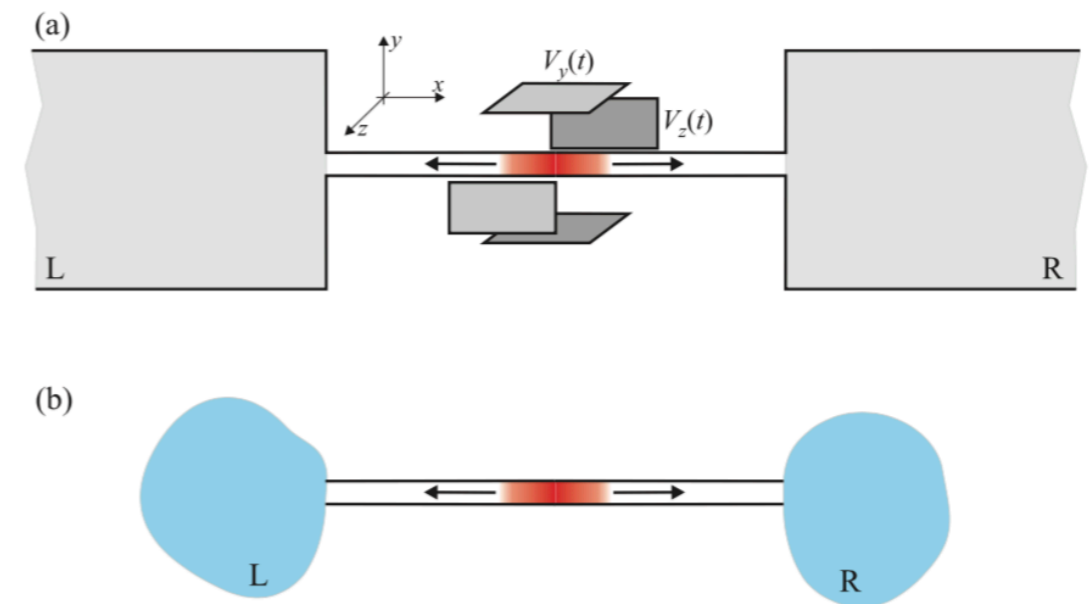


FIG. 1: Schematic visualizations of devices proposed in the text. (a) A spin-orbit-active weak link connects two contacts, L and R , to form a closed circuit. The time-dependent spin-orbit interaction is generated by two perpendicular gates, whose potentials $V_y(t)$ and $V_z(t)$ oscillate slowly in time with frequency Ω . The arrows within the weak link indicate the directions in which polarized electron-spins are flowing. (b) An open-circuit version of (a) where spin is accumulated in two terminals leading to a magnetization that can be measured.

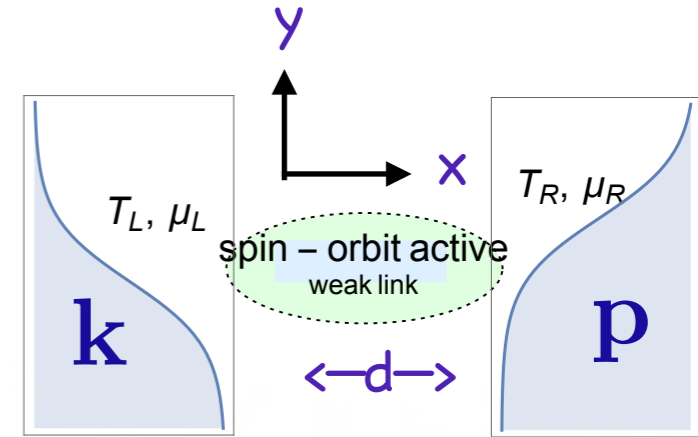
Jonson, Shekhter, OEW, Aharony, Park, Radic, DC spin generation by junctions with Ac driven spin-orbit interaction, arXiv:1903.03321



Time-dependent Rashba Hamiltonian (adiabatic approximation)

$$\mathcal{H}_{\text{tun}}(t) = \sum_{\mathbf{k}, \mathbf{p}} \sum_{\sigma, \sigma'} ([W_{LR}(t)]_{\sigma\sigma'} c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{p}\sigma'} + \text{H.c.})$$

$$W_{LR}(t) = J \exp[i\varphi_{AC}(t)] = J \exp[k_{\text{so}} d [\hat{\mathbf{x}} \times \hat{\mathbf{n}}(t)] \cdot \boldsymbol{\sigma}]$$



Circularly-rotating electric field

$$\hat{\mathbf{n}}(t) = \cos(\Omega t) \hat{\mathbf{z}} - \sin(\Omega t) \hat{\mathbf{y}}$$

elliptically-rotating electric field

$$\hat{\mathbf{n}}(t) = [\cos(\Omega t) \hat{\mathbf{z}} - \gamma \sin(\Omega t) \hat{\mathbf{y}}] / \sqrt{\cos^2(\Omega t) + \gamma^2 \sin^2(\Omega t)}$$

Validity of the adiabatic limit:

$$\Omega \ll [\textit{dwell time on link}]^{-1}$$

$$\hbar\Omega \lesssim \frac{G}{G_0} \frac{\hbar v_F}{d}$$

$$\textit{transparency} : \frac{G}{G_0} \sim 0.5$$

$$v_F \sim \frac{\ell_e e}{m^* \mu_e} \sim 10^8 \text{ cm/s for InAs nanowire}$$

$$\hbar\Omega \lesssim 3 \text{ meV} \Rightarrow \hbar\Omega \approx 0.1 \text{ meV}$$

$$\text{for } \Omega = 2\pi \times 20 \text{ GHz}$$



Details

System's Hamiltonian:

$$\mathcal{H} = \mathcal{H}_{\text{leads}} + \mathcal{H}_{\text{tun}}(t),$$

$$\mathcal{H}_{\text{tun}}(t) = \sum_{\mathbf{k}, \mathbf{p}} \sum_{\sigma, \sigma'} ([W_{LR}(t)]_{\sigma\sigma'} c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{p}\sigma'} + \text{H.c.})$$

$$\mathcal{H}_{\text{leads}} = \sum_{\mathbf{k}, \sigma} \epsilon_k c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} + \sum_{\mathbf{p}, \sigma} \epsilon_p c_{\mathbf{p}\sigma}^\dagger c_{\mathbf{p}\sigma}$$

“rate”: $[R_L(t)]_{\sigma\sigma'} = \frac{d}{dt} \sum_{\mathbf{k}} \langle c_{\mathbf{k}\sigma}^\dagger(t) c_{\mathbf{k}\sigma'}(t) \rangle$ Particle current (left lead): $I_L(t) = \sum_{\sigma} [R_L(t)]_{\sigma\sigma}$

Rate of change of the total spin (left lead):

$$\begin{aligned} \dot{\mathbf{M}}_L(t) &= \frac{d}{dt} \sum_{\sigma, \sigma'} \sum_{\mathbf{k}} \langle c_{\mathbf{k}\sigma}^\dagger(t) \boldsymbol{\sigma}_{\sigma\sigma'} c_{\mathbf{k}\sigma'}(t) \rangle \\ &= \sum_{\sigma, \sigma'} [R_L(t)]_{\sigma\sigma'} \boldsymbol{\sigma}_{\sigma\sigma'} \end{aligned}$$

$$\dot{M}_L^x(t) = \frac{d}{dt} \sum_{\mathbf{k}} \langle c_{\mathbf{k}\uparrow}^\dagger(t) c_{\mathbf{k}\downarrow}(t) + c_{\mathbf{k}\downarrow}^\dagger(t) c_{\mathbf{k}\uparrow}(t) \rangle$$

example:

$$\dot{M}_L^z(t) = \frac{d}{dt} \sum_{\mathbf{k}} \langle c_{\mathbf{k}\uparrow}^\dagger(t) c_{\mathbf{k}\uparrow}(t) - c_{\mathbf{k}\downarrow}^\dagger(t) c_{\mathbf{k}\downarrow}(t) \rangle$$

in units of: $g\mu_B/2$



Details

System's Hamiltonian:

$$\mathcal{H} = \mathcal{H}_{\text{leads}} + \mathcal{H}_{\text{tun}}(t),$$

$$\mathcal{H}_{\text{tun}}(t) = \sum_{\mathbf{k}, \mathbf{p}} \sum_{\sigma, \sigma'} ([W_{LR}(t)]_{\sigma\sigma'} c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{p}\sigma'} + \text{H.c.})$$

$$\mathcal{H}_{\text{leads}} = \sum_{\mathbf{k}, \sigma} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} + \sum_{\mathbf{p}, \sigma} \epsilon_{\mathbf{p}} c_{\mathbf{p}\sigma}^\dagger c_{\mathbf{p}\sigma}$$

“rate”:

$$[R_L(t)]_{\sigma\sigma'} = \frac{d}{dt} \sum_{\mathbf{k}} \langle c_{\mathbf{k}\sigma}^\dagger(t) c_{\mathbf{k}\sigma'}(t) \rangle$$

$$[R_L(t)]_{\sigma\sigma'} = \sum_{\mathbf{k}, \mathbf{p}} [f_R(\epsilon_{\mathbf{p}}) - f_L(\epsilon_{\mathbf{k}})] \int_{-\infty}^t dt_1 e^{\eta t_1}$$

$$f_L(\epsilon_{\mathbf{k}}) = \left(\exp\left[\frac{\epsilon_{\mathbf{k}} - \mu_L}{k_B T}\right] + 1 \right)^{-1}$$

$$\times \left(e^{i(\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{p}})(t - t_1)} [W_{LR}(t) W_{LR}^\dagger(t_1)]_{\sigma'\sigma} + \text{H.c.} \right), \quad \eta \rightarrow 0^+$$



Results:

Particle current is conserved: $I_L + I_R = 0$

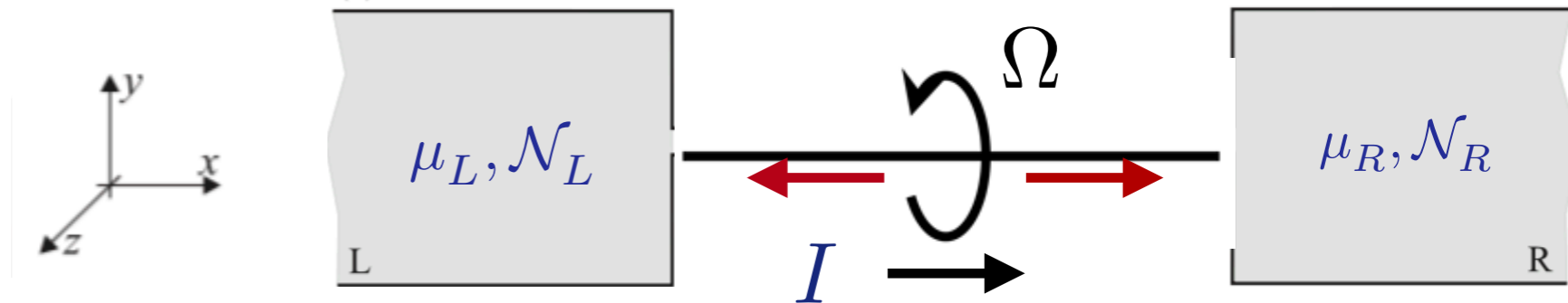
spin currents are

not: $\dot{M}_L(t) = \dot{M}_R(t)$

$$\dot{M}_L^x = \frac{G}{G_0} \mathcal{F}(\Omega) \sin^2(k_{so}d)$$

$$\mathcal{F}(\Omega) = \int \frac{d\omega d\omega'}{2\pi} [f_L(\omega) - f_R(\omega')] [\delta(\omega - \omega' + \Omega) - \delta(\omega - \omega' - \Omega)]$$

junction conductance (units of quantum conductance) G/G_0



Particle current:

$$I = \left(4\pi |W_0|^2 \mathcal{N}_L \mathcal{N}_R \right) (\mu_R - \mu_L)$$

junction conductance/e

DC longitudinal spin generated by AC electric field

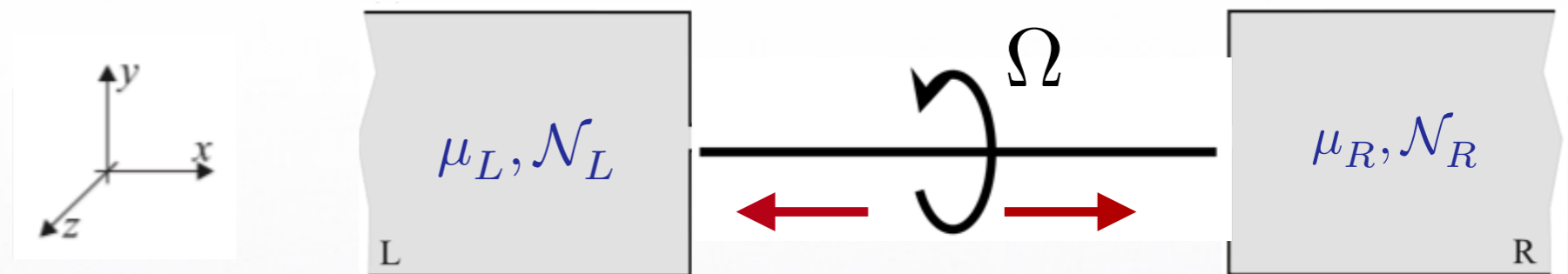


Transversed magnetization:

The sum of the two transversed spin component is along the vector

$$[0, \sin(\Omega t), -\cos(\Omega t)]$$

$$\dot{\mathbf{M}}_L^{\text{tr}}(t) = \frac{G}{G_0} \frac{\mathcal{F}(\Omega)}{2} \sin(2k_{\text{so}}d) [0, \sin(\Omega t), -\cos(\Omega t)]$$



$$\mathcal{F}(\Omega) = \int \frac{d\omega d\omega'}{2\pi} [f_L(\omega) - f_R(\omega')] [\delta(\omega - \omega' + \Omega) - \delta(\omega - \omega' - \Omega)]$$

junction conductance (units of quantum conductance) G/G_0

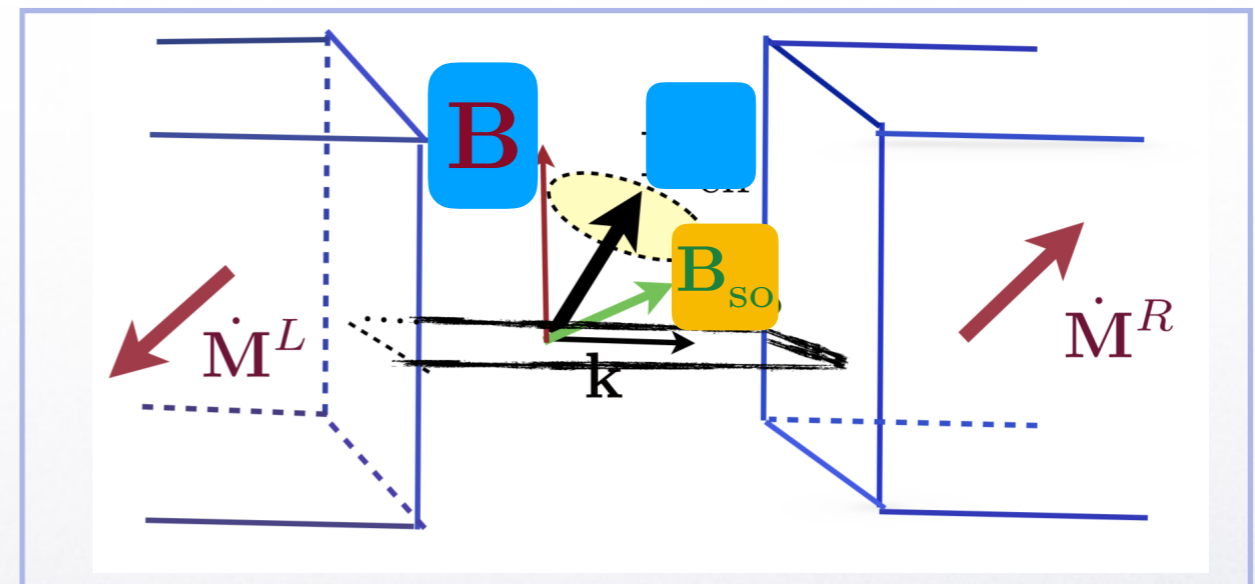


An un-biased weak link between two terminals, which is subjected to a Rashba spin-orbit interaction caused by an AC electric field that rotates periodically in the plane perpendicular to the link, injects spin polarized electrons into the terminals. The DC component of the polarization vanishes for a linearly-polarized electric field.

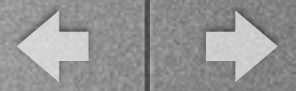


Breaking time-reversal symmetry with a Zeeman field

Due to the Zeeman field, both charge and spin current exhibit oscillations with the link's length in conjunction with the spin-orbit coupling. This can be used to measure the strength of the spin-orbit interaction



Aharony, OEW, Jonson, Shekhter, Electric and magnetic gating of Rashba-active weak links, PRB(R) 97, (2018)



tunneling amplitude—
the propagator

propagation of a plane wave
(wave vector k) along a
straight segment of length s

Shabhazyan and Raikh, Low-field anomaly in 2D hopping
magnetoresistance caused by spin-orbit term in the
energy spectrum, PRL 73, (1994)

$$P(E) = \int dk e^{iks} [E_F - \mathcal{H}(k)]^{-1}$$

$$\mathcal{H} = \frac{1}{2m^*} \left(-i \frac{d}{ds} - \frac{e}{c} \mathbf{A} \right)^2 + \frac{\tilde{k}_{so}}{m^*} \hat{\mathbf{n}} \cdot \boldsymbol{\sigma} \times \left(-i \frac{d}{ds} - \frac{e}{c} \mathbf{A} \right) - \mathbf{B} \cdot \boldsymbol{\sigma} \Rightarrow \mathcal{H}(k) = \frac{k^2}{2m^*} + \frac{k k_{so}}{m^*} (\hat{\mathbf{n}} \times \hat{\mathbf{s}}) \cdot \boldsymbol{\sigma} - \mathbf{B} \cdot \boldsymbol{\sigma}$$

omit Aharonov-Bohm phase due to \mathbf{A} , ignore
~~Zeeman interaction due to \mathbf{B}~~ , assume $\mathbf{n} \parallel \mathbf{E}$
normal to plane where k is



propagator
without time-reversal symmetry

$$\mathbf{H}_{\text{eff}}(\mathbf{k}) = \frac{\hbar k_{\text{so}}}{m^*} \hat{\mathbf{n}} \times \hat{\mathbf{s}} - \mathbf{B}$$

$$E \rightarrow k_{\text{F}}^2 / (2m^*) \quad G(s; E) = \int dk e^{iks} \frac{E + i0^+ - \frac{k^2}{2m^*} - \mathbf{H}_{\text{eff}}(\mathbf{k}) \cdot \boldsymbol{\sigma}}{(E + i0^+ - \frac{k^2}{2m^*})^2 - H_{\text{eff}}^2(\mathbf{k})}$$

Cauchy integration leads to two poles

$$k_{\pm}^2 - k_{\text{F}}^2 = \pm 2m^* H_{\text{eff}}(k_{\pm}) \quad \text{two residues} \quad A_{\pm}$$

\Rightarrow two polarizations

\Rightarrow Interference in spin space due to Aharonov Casher phase

\Rightarrow Tunneling amplitude is no more unitary (no more a simple rotation)

$$G(s; E) \propto e^{ik_+ s} A_+ (1 + \hat{\mathbf{q}}_+ \cdot \boldsymbol{\sigma}) + e^{ik_- s} A_- (1 - \hat{\mathbf{q}}_- \cdot \boldsymbol{\sigma})$$

$$\hat{\mathbf{q}}_{\pm} = \mathbf{H}_{\text{eff}}(k_{\pm}) / H_{\text{eff}}(k_{\pm})$$

$$\mathbf{B} = 0 \Rightarrow \hat{\mathbf{q}}_+ = \hat{\mathbf{q}}_- = \hat{\mathbf{n}} \times \hat{\mathbf{s}}$$



propagator
without
time-
reversal
symmetry

$$G(s; E) \propto e^{ik_+s} A_+ (1 + \hat{\mathbf{q}}_+ \cdot \boldsymbol{\sigma}) + e^{ik_-s} A_- (1 - \hat{\mathbf{q}}_- \cdot \boldsymbol{\sigma})$$

$$\hat{\mathbf{q}}_{\pm} = \mathbf{H}_{\text{eff}}(k_{\pm}) / H_{\text{eff}}(k_{\pm})$$

The two terms correspond to waves with wave

numbers k_+ and k_- (small Zeeman energy compared to Fermi

energy). The corresponding tunneling amplitudes contain

the spin projection matrices

$$1 \pm \hat{\mathbf{q}}_{\pm} \cdot \boldsymbol{\sigma}$$

The transmitted electrons are fully polarized along

q_+ and q_-

The non-unitarity of the propagator is due to the Zeeman field. Then a bias voltage between the leads generates charge and spin currents.



Hamiltonian

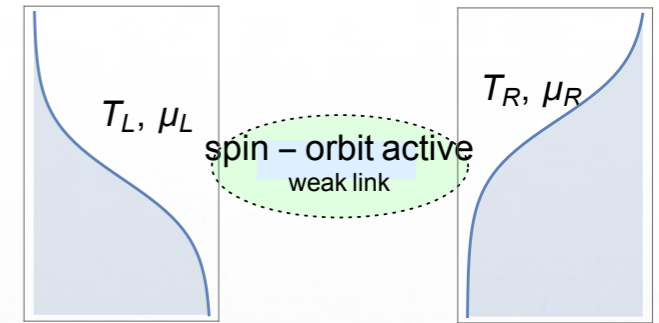
$$\mathcal{H} = \mathcal{H}_{\text{leads}} + \mathcal{H}_{\text{tun}}$$

$$\mathcal{H}_{\text{leads}} = \sum_{\mathbf{k}, \sigma} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} + \sum_{\mathbf{p}, \sigma} \epsilon_{\mathbf{p}} c_{\mathbf{p}\sigma}^\dagger c_{\mathbf{p}\sigma}$$

rate: $R_{\sigma\sigma'}^L = \frac{d}{dt} \sum_{\mathbf{k}} \langle c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma'} \rangle$

$$\mathcal{H}_{\text{tun}} = \sum_{\mathbf{k}, \mathbf{p}} \sum_{\sigma\sigma'} ([V_{\mathbf{k}\mathbf{p}}]_{\sigma\sigma'} c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{p}\sigma'} + \text{H.c.})$$

$$= 2\pi \sum_{\mathbf{k}, \mathbf{p}} \underbrace{[V_{\mathbf{k}\mathbf{p}} V_{\mathbf{k}\mathbf{p}}^\dagger]_{\sigma'\sigma}}_{\text{Both generated by the bias}} \delta(\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{p}}) \underbrace{[f_L(\epsilon_{\mathbf{k}}) - f_R(\epsilon_{\mathbf{p}})]}_{\text{Both generated by the bias}}$$



Particle current $I_L = \sum_{\sigma} R_{\sigma\sigma}^L$

Both generated by the bias

Spin current $\dot{M}_L = \sum_{\sigma, \sigma'} R_{\sigma\sigma'}^L \sigma_{\sigma\sigma'}$

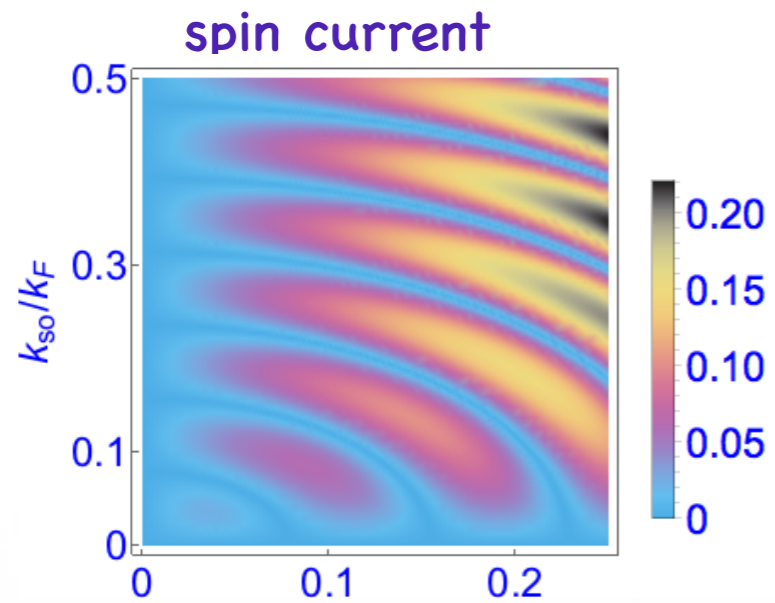
$$\mathcal{T}_{\sigma'\sigma} = \mathcal{T}_0 (U \delta_{\sigma, \sigma'} + \mathbf{W} \cdot [\boldsymbol{\sigma}]_{\sigma'\sigma})$$

charge current

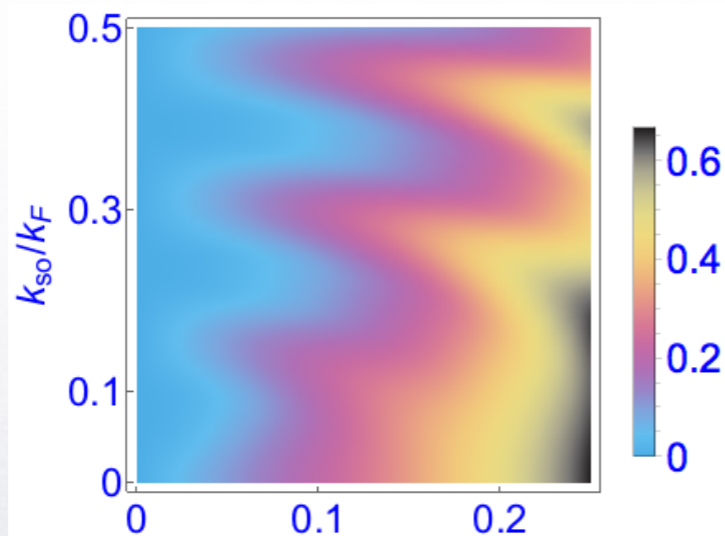
spin current



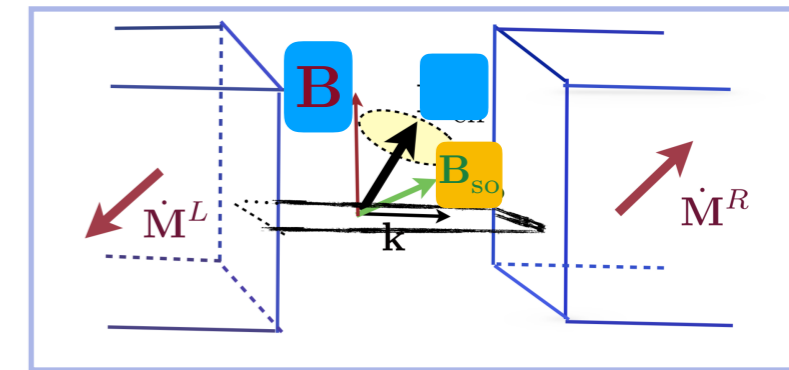
$$\mathbf{B} \perp \mathbf{B}_{\text{SO}}$$



in $\{\mathbf{B}_{\text{SO}}, \mathbf{B}\}$ - plane



normal to
 $\{\mathbf{B}_{\text{SO}}, \mathbf{B}\}$ - plane



magnetoconductance
oscillates with the length
of the weak link

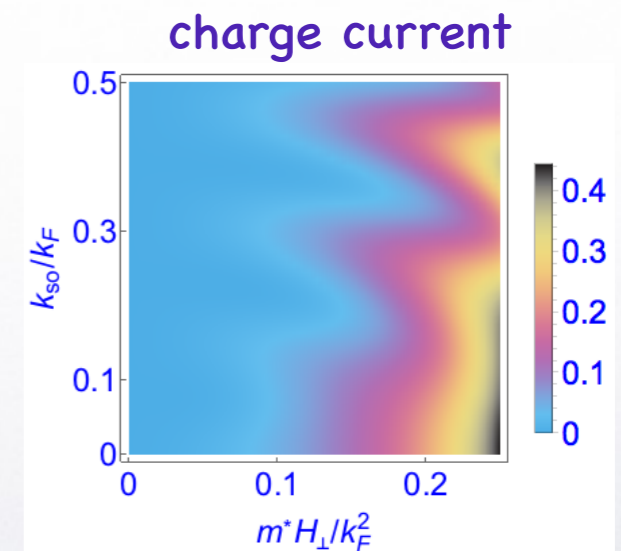


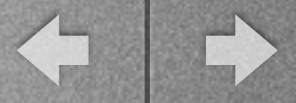
FIG. 2: (Color online.) The magnetoconductance difference, $U_{ii} - U_i$, calculated for $k_F d = 20$, as function of the spin-orbit coupling (k_{so}) measured in units of the Fermi wave vector, and the Zeeman energy, measured in units of k_F^2/m^* . The oscillations shown are due to the term $\propto \cos(\alpha)$ of U_{ii} ; $\alpha = (k_+ - k_-)d$.



*net amount of charge and magnetic moment per unit time is transferred through a biased, spin-orbit active, weak link;

*oscillations as function of $(k_+ - k_-)d$;

*the injected magnetization can be measured by a properly-positioned superconducting quantum interference device, or by a magnetic-resonance force microscope.



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